TYPE - I

02. $(D^2+6D+9)y = 5e^{3x}$

The given eqn is $(D^2+6D+9)y = 5e^{3x}$

Auxilary eqn is : $m^2+6m+9=0$

$$(m+3)(m+3)=0$$

$$m = -3, -3$$

The complementary function is

$$y_c = (Ax+B)e^{-3x}$$

particular integral is

$$y_p = \frac{1}{D^2+6D+9} 5e^{3x}$$

$$= \frac{1}{9+18+9} 5e^{3x}$$

$$= \frac{1}{36} 5e^{3x}$$

The complete solution is

$$y = y_c + y_p$$

$$y = (Ax+B)e^{-3x} + \frac{1}{36} 5e^{+3x}$$

06. $(D^2-6D+9)y = e^{3x}$

Auxilary eqn is $m^2-6m+9=0$

$$(m-3)(m-3)=0$$

$$m = 3, 3$$

The complementary function is

$$y_c = (Ax+B)e^{3x}$$

particular integral is $y_p = \frac{1}{D^2-6D+9} e^{3x}$

$$= \frac{1}{9-18+9} e^{3x}$$

$$= \frac{1}{0} e^{3x}$$

$$= \frac{x}{2D-6} e^{3x}$$

$$= \frac{x}{6-6} e^{3x} = \frac{1}{0} e^{3x}$$

$$y_p = \frac{x^2}{2} e^{3x}$$

The complete solution is

$$y = y_c + y_p$$

$$= (Ax+B)e^{3x} + \frac{x^2}{2} e^{3x}$$

7. $(D^2-4D+4)y = e^{2x}$

Auxilary eqn is $m^2-4m+4=0$

$$(m-2)(m-2)=0$$

$$m = 2, 2$$

The complementary eqn is

$$y_c = (Ax+B)e^{2x}$$

Particular integral is $y_p = \frac{1}{D^2-4D+4} e^{2x}$

$$= \frac{1}{4-8+4} e^{2x}$$

$$= \frac{x}{2D-4} e^{2x}$$

$$= \frac{x}{4-4} e^{2x} = \frac{1}{0} e^{2x}$$

$$= \frac{x^2}{2} e^{2x}.$$

The complete solution is

$$y = (Ax+B) e^{2x} + \frac{x^2}{2} e^{2x}$$

$\boxed{\text{TYPE - II}}$

4. $(D^2+4)\, y = 2\sin 2x.$

Auxilary eqn is $m^2+4 = 0 \Rightarrow m^2 = -4$

$$m = \pm 2i$$

The Complementary function is

$$y_c = A\cos 2x + B\sin 2x$$

The particular integral is

$$y_p = \frac{1}{D^2+4} 2\sin 2x$$

$$= \frac{1}{-4+4} 2\sin 2x.$$

$$= \frac{x}{\cancel{2}D} \cancel{2}\sin 2x$$

$$= \frac{x}{D} \sin 2x$$

$$= x\left[-\frac{\cos 2x}{2}\right]$$

$$= -\frac{x\cos 2x}{2}$$

The Complete solution is

$$y = y_c + y_p$$

$$= A\cos 2x + B\sin 2x - \frac{x}{2}\cos 2x$$

6. $(D^3 - 3D^2 + 4D - 2)\, y = \cos x.$

Auxilary eqn is $m^3 - 3m^2 + 4m - 2 = 0.$

$$(m-1)(m^2 - 2m + 2) = 0$$

$\boxed{m=1}$ $\quad m^2 - 2m + 2 = 0$

$$m = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$\boxed{m = (1 \pm i)}$$

Complementary function is

$$y_c = Ae^x + e^x(A\cos x + B\sin x)$$

Particular integral

$$y_p = \frac{1}{D^3 - 3D^2 + 4D - 2} \cos x$$

$$= \frac{1}{D\left(D^2 - 3D + 4 - \frac{2}{D}\right)} \cos x.$$

$$= \frac{1}{D\left(-1 - 3D + 4 - \frac{2}{D}\right)} \cos x$$

$$= \frac{1}{D\left(-3D + 3 - \frac{2}{D}\right)} \cos x$$

$$= \frac{1}{-3D^2 + 3D - 2} \cos x$$

$$= \frac{1}{-3(-1) + 3D - 2} \cos x = \frac{1}{3D+1} \cos x.$$

$$= \frac{(3D-1)}{(3D+1)(3D-1)} \cos x$$

$$= \frac{(3D-1)\cos x}{9D^2-1}$$

$$= \frac{3D(\cos x) - \cos x}{9(-1)-1}$$

$$= \frac{-3\sin x - \cos x}{-10}$$

$$y_P = \frac{3\sin x + \cos x}{10}$$

The complete solution is

$$y = Ae^x + e^x(B\cos x + C\sin x) + \frac{1}{10}(3\sin x + \cos x)$$

10) $(D^4 - 2D^3 + D^2)y = \cos x$

Auxilary eqn is $m^4 - 2m^3 + m^2 = 0$

$$m^2(m^2 - 2m + 1) = 0$$

$m^2 = 0$

$m = 0, 0$ $\quad (m-1)(m-1) = 0$

$m = 1, 1.$

Complementary eqn is

$$y_c = (Ax+B) + (Cx+D)e^x$$

Particular integral $y_p = \frac{1}{D^4 - 2D^3 + D^2} \cos x.$

$$= \frac{1}{D^2(D^2-2D+1)} \cos x$$

$$= \frac{1}{(-1)(-1-2D+1)} \cos x$$

$$= -\frac{1}{-2D} \cos x$$

$$= \frac{1}{2} \sin x$$

Complete solution is $y = y_c + y_p$

$$y = Ax + B + (Cx+D)e^x + \frac{1}{2}\sin x.$$

(Type - II)

11. (a) $(D^2-4D+3y)y = \sin 3x \cos 2x$

(April 19) Auxilary equation is

$$m^2-4m+3=0$$

$$(m-3)(m-1)=0$$

$$m=3 \text{ or } 1$$

Complementary function

$$y_c = Ae^{3x}+Be^{x}$$

particular Integral

$$y_p = \frac{1}{(D^2-4D+3)} \sin 3x \cos 2x$$

$\frac{1}{2}\sin(A+B)-\sin(A-B)$

$= \sin A \cos B$

$$= \frac{1}{2}\frac{1}{(D^2-4D+3)}(\sin 5x - \sin x)$$

$$y_{p_1} = \frac{1}{2}\frac{1}{D^2-4D+3}\sin 5x$$

$$= \frac{1}{2}\frac{1}{-25-4D+3}\sin 5x$$

$$= \frac{1}{2}\frac{1}{(-4D-22)}\sin 5x$$

$$= -\frac{1}{2}\cdot\frac{1}{(4D+22)}\sin 5x$$

$$= -\frac{1}{4}\cdot\frac{1}{(2D+11)}\sin 5x$$

$$= -\frac{1}{4}\frac{(2D-11)}{(2D+11)(2D-11)}\sin 5x$$

$$= -\frac{1}{4}\frac{(2D-11)\sin 5x}{4D^2-121}$$

$$= -\frac{1}{4}\frac{2D(\sin 5x) - 11\sin 5x}{4(-25)-121}$$

$$= -\frac{1}{4}\frac{10\cos 5x - 11\sin 5x}{-100-121}$$

$$= -\frac{1}{4}\cdot\frac{10\cos 5x - 11\sin 5x}{-221}$$

$$y_{p_1} = \frac{1}{4}\left(\frac{10\cos 5x - 11\sin 5x}{221}\right)$$

$$y_{p_2} = -\frac{1}{2}\frac{1}{D^2-4D+3}\sin x$$

$$= -\frac{1}{2}\frac{1}{(-1-4D+3)}\sin x$$

$$= -\frac{1}{2}\frac{1}{(-4D+2)}\sin x$$

$$= +\frac{1}{2}\frac{1}{(4D-2)}\sin x$$

$$= \frac{1}{4}\cdot\frac{1}{(2D-1)}\sin x$$

$$= \frac{1}{4}\frac{(2D+1)}{(2D-1)(2D+1)}\sin x$$

$= \frac{1}{4} \frac{\ldots}{4D^2 - 1}$

$= \frac{1}{4} \cdot \frac{(-2\cos x + \sin x)}{4(-1) - 1}$

$= \frac{-2\cos x + \sin x}{4 \cdot (-5)}$

$= \frac{-(2\cos x - \sin x)}{-20}$

$y_{P_2} = \frac{2\cos x - \sin x}{20}$

Complete integral,

$y = y_c + y_{P_1} + y_{P_2}$

$= Ae^{3x} + Be^{x} + \frac{1}{4}\left(\frac{10\cos 5x - 11\sin 5x}{221}\right) + \frac{2\cos x - \sin x}{20}$

April 19

$\boxed{\text{Type-III}}$

(b) $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = x^2$

Given eqn can be written as

$(x^2D^2 - 3xD + 4)y = x^2$

put $x = e^z$ and $z = \log x$

we get

$x^2 \frac{d^2y}{dz^2} = D'(D'-1)y$

$x\frac{dy}{dx} = D'y$

$D'(D'-1)y - 3D'y + 4y = x^2$

$(D'(D'-1) - 3D' + 4)y = x^2$

$(D'^2 - D' - 3D' + 4)y = x^2$

$(D'^2 - 4D' + 4)y = x^2$

Auxilary equation.

$m^2 - 4m + 4 = 0$

$(m-2)(m-2) = 0$

$m = -2, -2$

Complementary function is

$y_c = (Ax + B)e^{-2x}$.

Particular Integral.

$y_p = \frac{1}{D'^2 - 4D' + 4} x^2$

$= \frac{1}{D'^2 - 4D' + 4} e^{2z}$

$= \frac{1}{4 - 8 + 4} e^{2z}$

$$= \frac{\cancel{z}}{2D' - 4} \cdot e^{2z}$$

$$= \frac{z}{2(2)-4} e^{2z} \Rightarrow \frac{z}{0} e^{2z}$$

$$= \frac{z^2}{2} e^{2z}$$

$$y_p = \frac{z^2}{2} e^{2z}$$

Complete integral

$$y = y_c + y_p$$

$$y = (Ax+B)e^{-2x} + \frac{z^2}{2} e^{2z}$$

April-18

11 (a) $(D^2-4)y = e^{2x}$ $\boxed{\text{Type-I}}$

Auxilary equation is $m^2 - 4 = 0$

$$m^2 = \sqrt{4}$$

$$m = \pm 2$$

$$m = +2, -2$$

Complementary equation.

$$y_c = Ae^{2x} + Be^{-2x}.$$

particular integral.

$$y_p = \frac{1}{D^2-4} e^{2x} = \frac{1}{4-4} e^{2x}$$

$$= \frac{x}{2D} e^{2x}$$

$$= \frac{x}{4} e^{2x}$$

Complete integral is

$$y = y_c + y_p$$

$$y = Ae^{2x} + Be^{-2x} + \frac{x}{4} e^{2x}$$

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(b) $(D^2+3D+2)y = x^2$ $\boxed{\text{Type - III}}$

Auxilary eqn is $m^2+3m+2=0$

$$(m+1)(m+2) = 0$$

$$m = -1, -2$$

Complementary function $y_c = Ae^{-x} + Be^{-2x}$

Particular integral. $y_p = \frac{1}{D^2+3D+2} x^2$

$$= \frac{1}{2} \frac{1}{\left[1 + \left(\frac{D^2+3D}{2}\right)\right]} x^2$$

$$= \frac{1}{2}\left[1 + \left(\frac{D^2+3D}{2}\right)\right]^{-1} x^2$$

$$= \frac{1}{2}\left[1 - \left(\frac{D^2+3D}{2}\right) + \left(\frac{D^2+3D}{2}\right)^2 - \dots\right] x^2$$

$$= \frac{1}{2}\left(1 - \frac{D^2}{2} - \frac{3D}{2} + \frac{D^4}{4} + \frac{9D^2}{4} + \frac{6D^3}{4}\right) x^2$$

$$= \frac{1}{2}\left(x^2 - \frac{D^2(x^2)}{2} - \frac{3D(x^2)}{2} + \frac{9D^2(x^2)}{4} \cancel{+ 6D^3(x^2)}\right)$$

$$= \frac{1}{2}\left(x^2 - \frac{2}{2} - \frac{3(2x)}{2} + \frac{9(2)}{4}\right)$$

$$= \frac{1}{2}\left(x^2 - 1 - 3x + \frac{9}{2}\right)$$

$-1 + \frac{9}{2}$

$\frac{-2+9}{2}$

$y_p = \frac{1}{2}\left(x^2 - 3x + \frac{7}{2}\right)$

$y_p = \frac{1}{4}(2x^2 - 6x + 7)$

Complete solution

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$$y = Ae^{-x} + Be^{-2x} + \frac{1}{4}(2x^2 - 6x + 7)$$

$\boxed{\text{Type-5}}$

16. $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$ (10 Marks)

Put $x = e^z$ and $z = \log x$

$x^2\frac{d^2y}{dx^2} = D'(D'-1)y$

$x\frac{dy}{dx} = D'y$

$D'(D'-1)y - D'y - 3y = e^{2z} z$

$(D'^2 - D' - D' - 3)y = e^{2z} z$

$(D'^2 - 2D' - 3)y = e^{2z} z$

Auxliary equation is

$m^2 - 2m - 3 = 0$

$(m+1)(m-3) = 0$

$m = -1, 3$

Complementary equation is

$y_c = Ae^{-z} + Be^{3z}$

Particular Integral

$$y_p = \frac{1}{D'^2 - 2D' - 3} e^{2z} \cdot z$$

$$= e^{2z} \frac{1}{(D'+2)^2 - 2(D'+2) - 3} \cdot z$$

$$= e^{2z} \frac{1}{D'^2 + \cancel{4} + 4D' - 2D' - \cancel{4} - 3} \cdot z$$

$$= e^{2z} \frac{1}{D'^2 + 2D' - 3} z.$$

$$= e^{2z} \frac{1}{-3\left(1 - \left[\frac{D'^2 + 2D'}{3}\right]\right)} z$$

$$= \frac{-e^{2z}}{3}\left[1-\left(\frac{D'^2+2D'}{3}\right)\right]^{-1} z$$

$$= \frac{-e^{2z}}{3}\left(1+\frac{D'^2+2D'}{3}+\left(\frac{D'^2+2D}{3}\right)^2+\dots\right) z$$

$$= \frac{-e^{2z}}{3}\left(z+\frac{D'^2(z)}{3}+\frac{2D'(z)}{3}\right)$$

$$= \frac{-e^{2z}}{3}\left(z+\frac{2}{3}\right)$$

$$= \frac{-e^{2z}}{3}\left(\frac{3z+2}{3}\right)$$

$$= -\frac{x^2}{9}(3\log x+2)$$

Complete solution

$$y = Ae^{-z}+Be^{3z}-\frac{e^{2z}}{9}(3z+2)$$

$$\boxed{y = \frac{A}{x}+Bx^3-\frac{x^2}{9}(3\log x+2)}$$

April 19 16. $(D^2-4D+3)y = e^{-x}\sin x$ Type-IV

Auxilary eqn $m^2-4m+3=0$

$(m-1)(m-3)=0$

$m = 1, 3$

Complementary equatin is $y_c = Ae^{x}+Be^{3x}$

Particular Integral.

$$y_p = \frac{1}{D^2-4D+3}e^{-x}\sin x$$

$$= e^{-x}\frac{1}{(D-1)^2-4(D-1)+3}\sin x$$

$$= e^{-x}\frac{1}{D^2+1-2D-4D+4+3}\sin x$$

$$= e^{-x}\frac{1}{D^2-6D+8}\sin x$$

$$= e^{-x}\frac{1}{-1-6D+8}\sin x$$

$$= e^{-x}\frac{1}{-6D+7}\sin x$$

$$= e^{-x}\frac{1}{-(6D-7)}\sin x$$

$$= -e^{-x}\frac{(6D+7)}{(6D-7)(6D+7)}\sin x$$

$$= e^{-x} \frac{(6D+7)\sin x}{36D^2-49}$$

$$= e^{-x} \frac{6D(\sin x) + 7\sin x}{-36-49}$$

$$= \cancel{-} e^{-x} \frac{6\cos x + 7\sin x}{\cancel{-}85}$$

$$y_p = \frac{e^{-x}}{85}(6\cos x + 7\sin x).$$

Complete solution is

$$\boxed{y = Ae^{x} + Be^{3x} + \frac{e^{-x}}{85}(6\cos x + 7\sin x)}$$