$f(x_0)=0$ then x_0 is the root of the equation $f(x)=0$. Otherwise if $f(x_0) = -ve$ then the root lies between x_0 and a. Then as before the bisect the interval and the second approximation of the root is $x_1 = \frac{x_0+b}{2}$ or $\frac{x_0+a}{2}$

according the root lies between x_0 and b (or) x_0 and a.

This process continue till we get the root to our required accuracy.

Problem: 1

Find a real root of a equation $x^3-2x-5=0$. using the Bisection method?

<u>Sol:</u>

Given, $x^3-2x-5=0$.

Let $f(x) = x^3-2x-5$

$$f(0) = -5 = -ve.$$

$$f(1) = (1)^3-2(1)-5$$
$$= 1-2-5$$
$$= -6 = -ve.$$

$$f(2) = (2)^3-2(2)-5$$
$$= 8-4-5$$
$$= -1 = -ve.$$

$$f(3) = (3^3)-2(3)-5$$
$$= 27-6-5$$
$$= 16 = +ve.$$

The roots are lies between 2 & 3

$$x_0 = \frac{a+b}{2} \quad a=2, b=3$$

$$x_0 = \frac{2+3}{2} = \frac{5}{2}$$

$$\boxed{x_0 = 2.5}$$

$$f(x_0) = x_0^3 - 2x_0 - 5$$
$$= (2.5)^3 - 2(2.5) - 5$$
$$= 5.625 = +ve.$$

The roots are lies between 2 and 2.5

$$x_1 = \frac{a+b}{2}$$
$$= \frac{2+2.5}{2}$$
$$\boxed{x_1 = 2.25}$$

$x_1 = \frac{f(0) + x_0}{2}$ up to end of the follo $x_0 = x_1$

$$f(x_1) = x_1^3 - 2x_1 - 5$$
$$= (2.25)^3 - 2 \times 2.25 - 5$$
$$= 1.890625 = +ve.$$

The roots are lies between 2 and 2.25.

$$x_2 = \frac{a+b}{2}$$
$$= \frac{2+2.25}{2}$$
$$\boxed{x_2 = 2.125}$$

$$f(x_2) = x_2^3 - 2(x_2) - 5$$
$$= (2.125)^3 - 2(2.125) - 5$$
$$= 0.345703125 = +ve.$$

The roots are lies between 2 and 2.125

$$x_3 = \frac{a+b}{2}$$
$$= \frac{2+2.125}{2}$$
$$= 2.0625$$

$f(x_3) = (x_3)^3 - 2x_3 - 5$

$= (2.0625)^3 - 2(2.0625) - 5$

$= -0.351318359 = -ve$.

The roots are lies between 2.125 & 2.0625

$$x_4 = \frac{2.125 + 2.0625}{2}$$

$$\boxed{x_4 = 2.09375.}$$

$f(x_4) = (x_4)^3 - 2(x_4) - 5$

$= (2.09375)^3 - 2(2.09375) - 5$

$= -8.94165039 = -ve$.

The roots are lies between 2.125 & 2.09375

$$x_5 = \frac{2.125 + 2.09375}{2}$$

$$\boxed{x_5 = 2.109375.}$$

$f(x_5) = (x_5)^3 - 2(x_5) - 5$

$= (2.109375)^3 - 2(2.109375) - 5$

$= 0.16683578 4 = +ve$.

The roots are lies between 2.09375 & 2.109375

$$x_6 = \frac{a+b}{2}$$

$$= \frac{2.09375 + 2.109375}{2}$$

$$\boxed{x_6 = 2.1015625}$$

$f(x_6) = (x_6)^3 - 2(x_6) - 5$

$= (2.1015625)^3 - 2(2.1015625) - 5$

$= 0.078562259$

$= +ve$

The roots are lies between 2.09375 & 2.1015625.

$$x_7 = \frac{a+b}{2}$$

$$= \frac{2.09375 + 2.1015625}{2}$$

$$\boxed{x_7 = 2.09765625}$$

$$f(x_7) = (x_7)^3 - 2(x_7) - 5$$

$$= (2.09765625)^3 - 2(2.09765625) - 5$$

$$= 0.034714281 = +ve.$$

The roots are lies between 2.09375 & 2.09765625

$$x_8 = \frac{a+b}{2}$$

$$= \frac{2.09375 + 2.09765625}{2}$$

$$\boxed{x_8 = 2.095703125}$$

$$f(x_8) = (x_8)^3 - 2(x_8) - 5$$

$$= (2.095703125)^3 - 2(2.095703125) - 5$$

$$= 0.012862332 = +ve.$$

The roots are lies between 2.09375 & 2.095703125

$$x_9 = \frac{a+b}{2}$$

$$= \frac{2.09375 + 2.095703125}{2}$$

$$x_9 = 2.094726563$$

$$\boxed{x_9 = 2.095}$$

Problem : 2

Find the real root of equation $x^3 - x - 4 = 0$ using the Bisection method?

Soln:

Given $x^3 - x - 4 = 0$.

let $f(x) = x^3 - x - 4$

$f(0) = -4 = -ve$.

$f(1) = 1^3 - 1 - 4$

$= -4 = -ve$.

$f(2) = 2^3 - 2 - 4$

$= 8 - 2 - 4$

$= 2 = +ve$.

The roots are lies between 1 and 2.

$x_0 = \frac{a+b}{2}$; $a = 1$ $b = 2$.

$= \frac{1+2}{2} = \frac{3}{2}$

$\boxed{x_0 = 1.5}$

$f(x_0) = x_0^3 - x_0 - 4$

$= (1.5)^3 - 1.5 - 4$

$= -2.125 = -ve$.

The roots are lies between 2 and 1.5

$x_1 = \frac{a+b}{2}$; $a = 2$ $b = 1.5$

$= \frac{2+1.5}{2} = 1.75$

$\boxed{x_1 = 1.75}$

$f(x_1) = x_1^3 - x_1 - 4$

$= (1.75)^3 - 1.75 - 4$

$= -0.390625 = -ve.$

The roots are lies between 2 and 1.75.

$$x_2 = \frac{a+b}{2} \; ; \; a = 2 \quad b = 1.75$$

$$= \frac{2+1.75}{2}$$

$$\boxed{x_2 = 1.875}$$

$$f(x_2) = (x_2)^3 - x_2 - 4$$

$$= (1.875)^3 - 1.875 - 4$$

$$= 0.716796875 = +ve.$$

The roots are lies between 1.75 and 1.875.

$$x_3 = \frac{a+b}{2} \; ; \; a = 1.75, b = 1.875$$

$$= \frac{1.75 + 1.875}{2}$$

$$\boxed{x_3 = 1.8125}$$

$$f(x_3) = (x_3)^3 - x_3 - 4$$

$$= (1.8125)^3 - 1.8125 - 4$$

$$= 0.141845703 = +ve.$$

The roots are lies between 1.75 and 1.8125

$$x_4 = \frac{a+b}{2} \; ; \; a = 1.75, \; b = 1.8125$$

$$= \frac{1.75 + 1.8125}{2}$$

$$\boxed{x_4 = 1.78125}$$

$$f(x_4) = (x_4)^3 - x_4 - 4$$

$$= (1.78125)^3 - 1.78125 - 4$$

$$= -0.129608154 = -ve$$

The roots are lies between 1.8125 & 1.78125

$$x_5 = \frac{1.8125 + 1.78125}{2}$$

$$\boxed{x_5 = 1.796875}$$

$f(x_5) = x_5^3 - x_5 - 4$

$= (1.796875)^3 - 1.796875 - 4$

$= 1.80270387 = +ve.$

The roots are lies between 1.78125 & 1.796875

$x_6 = \frac{a+b}{2}$

$= \frac{1.78125 + 1.796875}{2}$

$\boxed{x_6 = 1.7890625}$

$f(x_6) = x_6^3 - x_6 - 4$

$= (1.7890625)^3 - (1.7890625) - 4$

$= -0.062730312 = -ve.$

The roots are lies between 1.796875 & 1.7890625

$x_7 = \frac{a+b}{2}$

$= \frac{1.796875 + 1.7890625}{2}$

$\boxed{x_7 = 1.79296875}$

$f(x_7) = x_7^3 - x_7 - 4$

$= (1.79296875)^3 - 1.79296875 - 4$

$= -0.0290\ 45879 = -ve$

The roots are lies between 1.796875 & 1.79296875

$x_8 = \frac{a+b}{2}$

$= \frac{1.796875 + 1.79296875}{2}$

$x_8 = 1.794921875$

$\boxed{x_8 = 1.7949}$

$f(x_8) = x_8^3 - x_8 - 4$

$= (1.794921875)^3 - 1.794921875 - 4$

$= -1.807064004 = -ve$

The roots are lies between 1.796875 and 1.794921875

$x_9 = \frac{a+b}{2}$

$= \frac{1.796875 + 1.794921875}{2}$

$= 1.7958984380$

$\boxed{x_9 = 1.7959}$

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Problem 3

Find a real root of an equation $x^4 - x - 10 = 0$ using the bisection method.

Soln:

Given $x^4 - x - 10 = 0$.

Let $f(x) = x^4 - x - 10$

$f(0) = -10 = -ve$

$f(1) = (1)^4 - 1 - 10$

$= -10 = -ve$.

$f(2) = (2)^4 - 2 - 10$

$= 16 - 2 - 10$

$= 4 = +ve$.

The roots are lies between 1 and 2.

$x_0 = \frac{a+b}{2}$

$= \frac{1+2}{2}$

$= \frac{3}{2}$ $\boxed{x_0 = 1.5}$

$$f(x_0) = (x_0)^4 - x_0 - 10$$
$$= (1.5)^4 - 1.5 - 10$$
$$= -6.4375 = -ve.$$

The roots are lies between 2 and 1.5

$$x_1 = \frac{a+b}{2}$$
$$= \frac{2+1.5}{2}$$
$$\boxed{x_1 = 1.75}$$

$$f(x_1) = (x_1)^4 - x_1 - 10$$
$$= (1.75)^4 - 1.75 - 10.$$
$$= -2.3710 = -ve.$$

The roots are lies between 2 and 1.75.

$$x_2 = \frac{a+b}{2}$$
$$= \frac{2+1.75}{2}$$
$$\boxed{x_2 = 1.875}$$

$$f(x_2) = (x_2)^4 - x_2 - 10$$
$$= (1.875)^4 - (1.875) - 10$$
$$= 0.48461 = +ve.$$

The roots are lies between 1.75 & 1.875

$$x_3 = \frac{a+b}{2}$$
$$= \frac{1.75+1.875}{2}$$
$$\boxed{x_3 = 1.8125}$$

$$f(x_3) = (x_3)^4 - x_3 - 10$$
$$= (1.8125)^4 - 1.8125 - 10$$

$= -1.02024 = -ve$

The roots are lies between 1.875 & 1.8125.

$$x_4 = \frac{a+b}{2}$$

$$= \frac{1.875 + 1.8125}{2}$$

$$\boxed{x_4 = 1.84375}$$

$$f(x_4) = (x_4)^4 - x_4 - 10.$$

$$= (1.84375)^4 - (1.84375) - 10$$

$$= -0.28773 = -ve.$$

The roots are lies between 1.875 & 1.84375

$$x_5 = \frac{a+b}{2}$$

$$= \frac{1.875 + 1.84375}{2}$$

$$x_5 = 1.859375$$

$$\boxed{x_5 = 1.85}$$

The root of the given equation is 1.85.