Iteration method or method of Successive Approximation.

(ii)

Let $f(x) = 0$ be the given equation whose roots or to be determined. In this iteration method, first we write the given equation in the form $x = \phi(x)$.

Let $x = x_0$ be an initial approximation of the required root α.

then the first approximation x_1 is given by $x_1 = \phi(x_0)$.

The Second, third, etc....

Approximation are given by

$$x_2 = \phi(x_1)$$
$$x_3 = \phi(x_2)$$
$$x_4 = \phi(x_3)$$
$$\vdots \qquad \vdots$$
$$x_n = \phi(x_{n-1})$$

Hence x_n is the n^{th} iteration and the value of x_n gives the root of the given equation at the iteration.

Problem: 1

Find a real root of the equation $x^3 + x^2 - 1 = 0$. By using iteration method.

Soln:

Given $x^3 + x^2 - 1 = 0$

Let $f(x) = x^3 + x^2 - 1$

$f(0) = -1 = -ve$

$f(1) = 1 = +ve$

(2) The roots are lies between 0 & 1

Given $x^3 + x^2 - 1 = 0$

$x^2(x+1) - 1 = 0 \Rightarrow x^2(x+1) = 1$

$x^2 = \frac{1}{x+1}$

$\phi(x) = \boxed{x = \frac{1}{\sqrt{x+1}}}$

$\phi'(x) = \frac{1}{(x+1)^{1/2}}$

$= (x+1)^{-1/2}$

$x^n = n x^{n-1}$

$= -\frac{1}{2}(x+1)^{-1/2-1}$

$= -\frac{1}{2}(x+1)^{-3/2}$

$$\boxed{\phi'(x) = \frac{-1}{2(x+1)^{3/2}}}$$

$\boxed{|\phi'(x)| < 1}$ For condition

$|\phi'(0)| = \frac{-1}{2(0+1)^{3/2}} = \frac{-1}{2(1)^{3/2}}$

$= -0.25 < 1$

$|\phi'(1)| = \frac{-1}{2\cancel{\phi}(1+1)^{3/2}} = \frac{-1}{2(2)^{3/2}}$

$= -0.03 < 1$

$\therefore$ The condition is satisfied.

$x_0 = 0.75.$

The roots of the given equation is 0.7548.

(4) Problem : 2

Find the real root of the equation $\cos x = 3x - 1$. By using iteration method.

Soln:

Given $\cos x = 3x - 1 \Rightarrow \cos x - 3x + 1 = 0$.

Let $f(x) = \cos x - 3x + 1$

$f(0) = 1 - 0 + 1 = 2$ (+ve)

$f(1) = \cos 1 - 3 + 1$

$= -1.4596$ (−ve)

The roots lies between 0 and 1

Given $\cos x - 3x + 1 = 0$.

$\cos x - 3x = -1$

$$\phi(x) = \boxed{x = \frac{1 + \cos x}{3}}$$

$$\phi'(x) = \frac{-\sin x}{3}$$

$$|\phi'(0)| = \frac{-\sin 0}{3} = 0 < 1$$

$$|\phi'(1)| = \frac{-\sin 1}{3} = -0.28049 < 1$$

∴ The conditions are satisfied.

$x_0 = 0.75$

$$x_1 = \phi(x_0) = \frac{1 + \cos x_0}{3}$$

$$= \frac{1 + \cos(0.75)}{3}$$

$$\boxed{x_1 = 0.6666}$$

$$x_2 = \phi(x_1) = \frac{1+\cos x_1}{3}$$

$$= \frac{1+\cos(0.6666)}{3}$$

(5)

$$\boxed{x_2 = 0.6666}$$

(6)

The roots of the given equation is 0.6666

Problem : 3

Find the real root of the equation $\cos x - 3x + 2 = 0$. By using iteration method!

Soln:

Given $\cos x - 3x + 2 = 0$.

Let

$$f(x) = \cos x - 3x + 2$$

$$f(0) = \cos 0 - 3\times 0 + 2$$

$$= 3 = +ve$$

$$f(1) = \cos 1 - 3\times 1 + 2$$

$$= -1.5230 = -ve$$

The roots are lies between 0 & 1.

Given

$$\cos x - 3x + 2 = 0$$

$$-3x = -2 - \cos x$$

$$\phi(x) = \boxed{x = \frac{2+\cos x}{3}}$$

$$\phi'(x) = \frac{-\sin x}{3}$$

$$|\phi'(0)| = \frac{-\sin 0}{3} \quad 0 < 1$$

$$|\phi'(1)| = \frac{-\sin 1}{3} = -0.2804 < 1$$

∴ The condition is satisfied -

$x_0 = 0.75$

$$x_1 = \phi(x_0) = \frac{2+\cos x_0}{3} \quad x_n$$

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$$= \frac{2+\cos(0.75)}{3}$$

$$= 0.9999.$$

$$x_2 = \phi(x_1) = \frac{2+\cos x_1}{3}$$

$$= \frac{2+\cos(0.9999)}{3}$$

$$= 0.9999.$$

The roots of the given equation is 0.9999.

Problem: 4

Find a real root of the equation $x^2-5x+2=0$. By using iteration method.

Soln:

Given $x^2-5x+2=0$.

Let

$$f(x) = x^2-5x+2$$

$$f(0) = 0^2-5\times0+2 = 2 = +ve.$$

$$f(1) = 1^2-5(1)+2 = 1-5+2 = -2 = -ve$$

The roots are lies between 0 & 1.

Given

$$x^2-5x+2=0.$$

$$x(x-5)+2=0 \qquad x^2 = -2+5x.$$

$$x(x-5) = -2. \qquad x = \sqrt{}$$

$$\phi(x) = \boxed{x = \frac{-2}{x-5}}. \qquad \frac{u}{v} = \frac{vu'-uv'}{v^2}$$

$$\phi'(x) = \frac{(x-5)\cdot 0+2(1)}{(x-5)^2}.$$

$$\phi'(x) = \frac{2}{(x-5)^2}.$$

(7) $$|\phi'(0)| = \frac{2}{(0-5)^2} = \frac{2}{25} = 0.08 < 1$$

$$|\phi'(1)| = \frac{2}{(1-5)^2} = \frac{2}{(-4)^2} = \frac{2}{16} = 0.125 < 1$$

$\therefore$ The conditions are satisfied.

$x_0 = 0.75$

$$x_1 = \phi(x_0) = \frac{-2}{(x_0-5)} = \frac{-2}{(0.75-5)} = \boxed{0.4705}$$

$$x_2 = \phi(x_1) = \frac{-2}{(x_1-5)} = \frac{-2}{(0.4705-5)} = \boxed{0.4415}$$

$$x_3 = \phi(x_2) = \frac{-2}{(x_2-5)} = \frac{-2}{(0.4415-5)} = \boxed{0.4387}$$

$$x_4 = \phi(x_3) = \frac{-2}{(x_3-5)} = \frac{-2}{(0.4387-5)} = \boxed{0.4384}$$

The root of the given equation is 0.4384.

Newton Raphson method:

Newton formula is

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = Q(x_n)}$$

Problem: 1

Find the +ve root of the equation $x^3 - 3x + 1 = 0$. using newton Raphson method?

Soln:

Given $x^3 - 3x + 1 = 0$

Let $f(x) = x^3 - 3x + 1$

$f(0) = 1 = +ve.$

$f(1) = -1 = (-ve)$

The roots are lies between 0 & 1.

$$x_0 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

Using Bisec method

Given $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

Newton formula is

Apply Newton's formula

Using iteration method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Put $n = 0$,

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_0 - \frac{x^3 - 3x + 1}{3x^2 - 3}$$

$$= (0.5) - \frac{x_0^3 - 3x_0 + 1}{3(x_0)^2 - 3}$$

$$= 0.5 - \frac{(0.5)^3 - 3(0.5) + 1}{3(0.5)^2 - 3}$$

$$= 0.5 - \left[\frac{+0.375}{+2.25}\right]$$

$$= 0.5 - 0.16666$$

$$\boxed{x_1 = 0.33334}$$

Put $n = 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.33334 - \frac{x_1^3 - 3x_1 + 1}{3x_1^2 - 3}$$

$$= 0.33334 - \frac{(0.33334)^3 - 3(0.33334) + 1}{3(0.33334)^2 - 3}$$

$$= 0.33334 - \frac{0.03701}{-2.6666}$$

$$= 0.33334 - (-0.01387)$$

$$\boxed{x_2 = 0.34721}$$

Put $n = 2$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.34721 - \frac{x_2^3 - 3x_2 + 1}{3x_2^2 - 3}$$

$$= 0.34721 - \frac{(0.34721)^3 - 3(0.34721) + 1}{3(0.34721)^2 - 3}$$

$$= 0.34721 - \frac{\cancel{2.2782}\ 0.00022}{-2.6383}$$

$$= 0.34721 - (\cancel{-8.85870})$$

$$(-0.000086039)$$

$$\boxed{x_3 = 0.34721}$$

Problem : 2

Find the +ve roots of the equation $x^3 - 2x + 0.5 = 0$. Busing newton Raphson method?

Soln:

Given $x^3 - 2x + 0.5 = 0$.

Let

$$f(x) = x^3 - 2x + 0.5$$

$$f(0) = 0^3 - 2(0) + 0.5 = 0.5 = +ve.$$

$$f(1) = 1^3 - 2(1) + 0.5 = 1 - 2 + 0.5$$

$$= -0.5 = -ve.$$

The roots are lies between 0 and 1.

$$x_0 = \frac{a+b}{2} = \frac{0+1}{2} = \frac{1}{2} = 0.5.$$

Given

$$f(x) = x^3 - 2x + 0.5$$

$$f'(x) = 3x^2 - 2.$$

Newton formula is

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

Put $n = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.5 - \frac{(0.5)^3 - 2(0.5) + 0.5}{3(0.5)^2 - 2}$$

$$= 0.5 - \frac{(-0.375)}{(-1.25)}$$

$$= 0.5 - 0.3$$

$$\boxed{x_1 = 0.2}$$

Put $n=1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.2 - \frac{(0.2)^3 - 2(0.2) + 0.5}{3(0.2)^2 - 2}$$

$$= 0.2 - \frac{0.108}{-1.88}$$

$$= 0.2 - (-0.0574)$$

$$\boxed{x_2 = 0.2574}$$

Put $n=2$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.2574 - \frac{(0.2574)^3 - 2(0.2574) + 0.5}{3(0.2574)^2 - 2}$$

$$= 0.2574 - \frac{2.25397}{-1.80123}$$

$$= 0.2574 - (-1.25135)$$

$$x_3 = 1.50875$$