Problem: 2

Ans: 96.84

The population of a town in the decimal cens was as given below. estimate the population for the year 1925.

Year x:	1891	1901	1911	1921	1931
Population y:	46	66	81	93	101

Soln:

The difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

To find p.

$$x = x_n + Ph$$

$$x = 1925 \; ; \; x_n = 1931 \; ; h = 10.$$

$$1925 = 1931 + P(10)$$

$$P(10) = 1925 - 1931$$

$$P = \frac{1925 - 1931}{10}$$

$$\boxed{P = -0.6}$$

Newton's Backward formula is

$$y(1925) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n + \frac{P(P+1)(P+2)(P+3)}{4!}\nabla^4 y_n \cdots$$

$$y(1925) = 101 + (-0.6)(8) + \frac{(0.6)(0.6+1)}{2!}(-4)$$

$$+ \frac{(0.6)(0.6+1)(-0.6+2)}{3!}(-1) +$$

$$\frac{(-0.6)(0.6+1)(-0.6+2)(-0.6+3)}{4!}(-3)$$

$$= 101 + -4.8 + \frac{0.96}{2!} + \frac{0.336}{3!} + \frac{2.4192}{4!}$$

$$= 101 - 4.8 + 0.48 + 0.056 + 0.1008$$

$$\boxed{y(1925) = 96.84}$$

Problem : 3

The population of a town in the decimal cells was as given below. estimate the population for the year 1955.

year x :	1921	1931	1941	1951	1961
Population y :	46	66	81	93	101

Soln:

The difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1921	46				
		20			
1931	66		-5		
		15		2	
1941	81		-3		-3
		12		-1	
1951	93		-4		
		8			
1961	101				

To find p

$$x = x_n + ph$$

$x = 1955 \quad x_n = 1961 \quad h = 10$

$$x = x_n + Ph$$

$$1955 = 1961 + P(10)$$

$$P(10) = 1955 - 1961$$

$$P = \frac{1955 - 1961}{10}$$

$$\boxed{P = -0.6}$$

Newton's Backward formula is.

$$y(x) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\nabla^3 y_n$$

$$+ \frac{P(P+1)(P+2)(P+3)}{4!}\nabla^4 y_n.$$

$$= 101 + (-0.6)(8) + \frac{(-0.6)(-0.6+1)}{2!}(-4)$$

$$+ \frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(-1) + \frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{4!}(-3).$$

$$= 101 - 4.8 + \frac{0.96}{2!} + \frac{0.336}{3!} + \frac{2.4192}{4!}$$

$$= 101 - 4.8 + 0.48 + 0.056 + 0.1008.$$

$$= 96.836.$$

$$\boxed{y(1955) = 96.836}$$

Problem: 4

The function $f(x)$ is given by the following table find $f(0.2)$

x	0	1	2	3	4	5	6
$f(x)$	176	185	194	203	212	220	229.

soln:

The difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	176	9	0	0	0	-1	5
1	185	9	0	0	-1	4	
2	194	9	0	-1	3		
3	203	9	-1	2			
4	212	8	1				
5	220	9					
6	229						

$$x = x_0 + Ph$$

$$x = 0.2 \;;\; x_0 = 0 \;;\; h = 1$$

$$0.2 = 0 + P(1)$$

$$\boxed{P = 0.2}$$

Newton's forward formula is

$$y(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0$$

$$+ \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_0 + \frac{P(P-1)(P-2)(P-3)(P-4)}{5!}\Delta^5 y_0$$

$$+ \frac{P(P-1)(P-2)(P-3)(P-4)(P-5)}{6!}\Delta^6 y_0$$

$$= 176 + (0.2)(9) + \frac{0.2(0.2-1)}{2!}\times 0 +$$

$$\frac{0.2(0.2-1)(0.2-2)}{3!}\times 0 + \frac{0.2(0.2-1)(0.2-2)(0.2-3)}{4!}\times$$

$$+ \frac{0.2(0.2-1)(0.2-2)(0.2-3)(0.2-4)}{5!}(-1)$$

$$+ \frac{0.2(0.2-1)(0.2-2)(0.2-3)(0.2-4)(0.2-5)\times 5}{6!}$$

$= 176 + 1.8 + 0 + 0 + 0 + \frac{(-3.06432)}{5!} + \frac{(-73.54368)}{6!}$

$= 176 + 1.8 - 0.025536 - 0.102144$

$= 177.7$

$\boxed{f(0.2) = 177.7}$

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Problem 5:

In the table below the value of y are consecutive terms of a series of which the numbers · 21.6 is the 6th term. Find the 1st and 10th term series.

x	3	4	5	6	7	8	9.
y	2.7	6.4	12.5	21.6	34.3	51.2	72.9.

Soln:

y(1) The difference table is

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$	$\Delta^6 y_0$
3	2.7						
		3.7					
4	6.4		2.4				
		6.1		0.6			
5	12.5		3		0		
		9.1		0.6		0	
6	21.6		3.6		0		0.
		12.7		0.6		0	
7	34.5		4.2		0		
		16.9		0.6			
8	51.2		4.8				
		21.7					
9	72.9.						

$x = x_n + Ph$; $x = 1$; $x_9 = 9$, $h = 1$

$1 \pm 9 + P(1)$

$1 - 9 = P$

$$\boxed{P = -8}$$

$$y_n(x) = y_n + P \nabla y_n + \frac{P(P+1)}{2!} \nabla^2 y_n +$$

$$\frac{P(P+1)(P+2)}{3!} \nabla^3 y_n + \frac{P(P+1)(P+2)(P+3)}{4!} \nabla^4 y_n$$

$$+ \frac{P(P+1)(P+2)(P+3)(P+4)}{5!} \nabla^5 y_n + \frac{P(P+1)(P+2)(P+3)(P+4)(P+5)}{6!} \nabla^6 y_n$$

$$= 72.9 + (-8)(21.7) + \frac{(-8)(-8+1)}{2} +$$

$$\frac{(-8)(-8+1)(-8+2)(0.6)}{6} + \frac{(-8)(-8+1)(-8+2)(-8+3)(0)}{24}$$

$$+ \frac{(-8)(-8+1)(-8+2)(-8+3)(-8+4)(0)}{120} +$$

$$\frac{-8(-8+1)(-8+2)(-8+3)(-8+4)(-8+5) \times 0}{720}$$

$y(1) = 0.1$

y(10) The difference table is

x	y						
3	2.7						
		3.7					
4	6.4		2.4				
		6.1		0.6			
5	12.5		3		0		
		9.1		0.6		0	
6	21.6		3.6		0		0.
		12.7		0.6		0	
7	34.5		4.2		0		
		16.9.		0.6			
8	51.2.		4.8				
		21.7					
9	72.9.						

$x = x_0 + Ph$; $x = 10$; $x_0 = 3$; $h = 1$.

$10 = 3 + P(1)$

$P = 10 - 3 = 7$

$\boxed{P = 7}$

$$y_n(x) = y_0 + P\Delta y_n + \frac{P(P-1)}{2}\Delta^2 y_n + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_n$$

$$+ \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_n + \frac{P(P-1)(P-2)(P-3)(P-4)}{5!}\Delta^5 y_n$$

$$+ \frac{P(P-1)(P-2)(P-3)(P-4)(P-5)}{6!}\Delta^6 y_n + \cdots$$

$$= 2.7 + 7(3.7) + \frac{7(7-1)}{2}(2.4) + \frac{7(6)(5)(0.6)}{6} +$$

$$\frac{7(7-1)(7-2)(7-3)}{24} \times 0 + \frac{7(7-1)(7-2)(7-3)(7-4)}{120} \times 0$$

$$+ \frac{7(7-1)(7-2)(7-3)(7-4)(7-5)}{720} \times 0$$

$y(10) = 100$.

Lagrange's Interpolation formula for unequal intervals.

Let $f(x_0), f(x_1) \dots f(x_n)$ Be the value of the function $y = f(x)$ corresponding to the argument $x_0, x_1, \dots x_n$, not necessarily equally spaced.

Let $f(x)$ be a polynomial in x of degree 'n'. then we can represent the formula

$$f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} \times y_0$$

$$+ \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} \times y_1 +$$

$$\frac{(x-x_0)(x-x_1)\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)\dots(x_2-x_n)} \times y_2 +$$

$$\frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} \times y_n$$

Problem: 1

Using Lagrange's interpolation formula find the value corresponding to $x = 10$ from the following table.

x	5 x_0	6 x_1	9 x_2	11 x_3
y	12 y_0	13 y_1	14 y_2	16 y_3

Soln:

$x = 10$ $\quad x_0 = 5$ $\quad x_1 = 6$ $\quad x_2 = 9$ $\quad x_3 = 11$

$y_0 = 12$ $\quad y_1 = 13$ $\quad y_2 = 14$ $\quad y_3 = 16$ $\quad y = ?$

Formula is

$$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} \times y_0 +$$

$$\frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} \times y_1 + \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)\cdots(x_2-x_n)} \times$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} \times y_n .$$

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3 .$$

$$y(10) = \left[\frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} \times 12\right] +$$

$$\left[\frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} \times 13\right] +$$

$$\left[\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} \times 14\right] +$$

$$\left[\frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16\right]$$

$$= \left[\frac{4\times1\times1}{1\times(-4)\times(-6)}\times 12\right] + \left[\frac{5\times1\times(-1)}{1\times(-3)(-5)}\times 13\right]$$

$$+ \left[\frac{5\times4\times(-1)}{4\times3\times-2}\times 14\right] + \left[\frac{5\times4\times1}{6\times5\times2}\times 16\right]$$

$$= 14.666 = 14.67.$$

Problem: 2

Find the value of $f(x)$ are given below for certain value of x.

x	0	1	3	4
y	5	6	50	105

. Find the value of $f(2)$ using Lagrange's interpolation formula.

Sol:

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\times y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\cdot y_3$$

Here

$x=2$ $x_0=0$ $x_1=1$ $x_2=3$ $x_3=4$.

$$f(2) = \frac{(2-1)(2-3)(2-4)}{(0-1)(0-3)(0-4)}\cdot 5 + \frac{(2-0)(2-3)(2-4)}{(1-0)(1-3)(1-4)}\cdot 6$$

$$+ \frac{(2-0)(2-1)(2-4)}{(3-0)(3-1)(3-4)}\cdot 50 + \frac{(1-0)(2-1)(2-3)}{(4-0)(4-1)(4-3)}\cdot 105.$$

$$= \frac{(1)(-1)(-2)}{(-1)(-3)(-4)}(5) + \frac{(2)(-1)(-2)}{(1)(-2)(-3)}(6) +$$

$$\frac{(2)(1)(-2)}{(3)(2)(-1)}(50) + \frac{(2)(1)(-1)}{(4)(3)(1)} \cdot 105$$

$$= -0.8333 + 4 + 33.33 + 7.5$$

$$f(2) = 18.9967$$

$$= 19$$

Problem: 3

Using Lagrange's interpolation formula to find the value of y at $x = 3.5$ with the following data

x	1	3	5
y	1.	1.5719	1.5738

Soln:-

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2 +$$

Here

$x = 3.5$, $x_0 = 1$, $x_1 = 3$, $x_2 = 5$

$y_0 = 1.5708$, $y_1 = 1.5719$, $y_2 = 1.5738$

$$= \frac{(3.5-3)(3.5-5)}{(1-3)(1-5)} \times 1.5708 +$$

$$\frac{(3.5-1)(3.5-5)}{(3-1)(3-5)} \times 1.5719 +$$

$$\frac{(3.5-1)\ (3.5-3)}{(5-1)\ (5-3)} \times 1.5738$$

$$= \frac{(0.5)(-1.5)}{(-2)(-4)} \times 1.57080 + \frac{(2.5)(-1.5)}{(2)\ (-2)} \times 1.5719$$

$$+ \frac{(2.5)(0.5)}{(4)(2)} \times 1.5738$$

$$= -0.14726 + 1.47365 + 0.24590$$

$$= 1.57229$$