UNIT- I

(1) Introduction to operation Research :-

1. Operation Research :

A new approach to systematic and scientific study of the operations of a system is called the operation Research or operational research [abbreviated as OR]

2. Definition of OR :

OR is a scientific approach to problem solving for executive management by H.M. Wagner.

OR is the art of giving bad answers to problems which otherwise have worse answers, by T.S. Saaty.

OR is the application of scientific methods, techniques and tools to problems involving the operations of a system. So as to provide those in control of the system with optimum solutions to the problem by Churchman, Ackoff and Arnoff.

Mathematical formulation of LPP :

LPP - Linear Programming Problems.

1. Linear programming :-

Linear programming is a technique for determining an optimum schedule of inter-dependent activities in views of the available resources

2. Programming :-

Programming is just another word for planning and refers to the process of determining a particular plan of action from amongst several alternatives.

3. Linear :

The linear stands for indicating that all relationships involved in a particular problem or linear.

Mathematical formulation of the problem:

The procedure for Mathematical formulation of LPP :-

Step 1 :

Study the given situation to find the key decision to be made.

Step 2 :

Identify the variables involved and designate them by the symbol $x_i (j = 1, 2 \ldots)$

Step 3 :

State the feasible alternatives which generally are $x_j \geq 0 \; \forall j$

Step 4 :

Identify the constrains in the problem and express them as linear inequalities or equations LHS of which are linear function of the decision variables.

Step 5:

Identify the objectives function and express it as a linear function of the decision variables.

1. A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suitings, shirtings and woollen yieldings a profit of RS. 2, RS. 4 and RS. 3 permetre respectively. 1 m suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly 1 metre of shirtings requires 4 min in weaving, 1 min in processing and 3 min in packing. Similarly 3 min in each department in woollen. In a week total runtime of each department is 60, 40 and 80 hours for weaving. Processing and packing departments respectively formulate the linear programming problem To find the product next 15 minimize the profit.

≤ condition

z = eqn

max

≥

Soln:

4)

	Weaving in minutes	processing in minutes	packing in minutes	profit Rs
Suitings	3	2	1	2
Shirtings	4	1	3	4
Woollens	3	3	3	3
In weeks hours	60×60 = 3600 min	40×60 = 2400 min	80×60 = 4800 min	

Step 1:

The key decision is to determine the weekly rate of production for the three types of clothes.

Step 2:

Let us designate the weekly production of Shirtings, Suitings and woollens x_1m, x_2m and x_3 metres respectively

Step 3:

Since it is not possible to produce negative quantities, feasible alternatives are sets of values of x_1, x_2 and x_3 satisfying $x_1 \geq 0$, $x_2 \geq 0$ and $x_3 \geq 0$.

Step 4:

The constraints are limited availability of three operational departments 1m of Suitings requires 3 min of weaving. The quantity being x_1 metres. The requirements of suitings along will be $3x_1$ units.

Similarly x_2 meters of shirtings and x_3 metres of woollen will be require $4x_2$ and $3x_3$ minutes respectively.

Thus the total requirements of weaving will be $3x_1 + 4x_2 + 3x_3$ which should not exceed the available 3600 minute. The labour constrain becomes $3x_1 + 4x_2 + 3x_3 \leq 3600$.

x_1 meters of suitings, x_2 metres of shirting and x_3 meters of woollen will require $2x_1$; x_2 and $3x_3$ minutes respectively.

The total requirements of processing will be $2x_1 + x_2 + 3x_3$ which should not exceed the available 2400 minutes. The labour constrain becomes $2x_1 + x_2 + 3x_3 \leq 2400$

x_1 meters of suitings, x_2 meters of shirting and x_3 meters of woollen will require x_1, $3x_2$, $3x_3$ minutes respectively

The total requirements of packing will be $x_1 + 3x_2 + 3x_3$ which should not exceed the available 4800 minutes. The labour constrain becomes $x_1 + 3x_2 + 3x_3 \leq 4800$.

Step 5:

The objective to minimize the total profit from sales assuming that product is sold in the market. Therefore the total profit is given by the linear maximum $z = 2x_1 + 4x_2 + 3x_3$

$\therefore$ The general form of LPP

1. Maximize $z = 2x_1 + 4x_2 + 3x_3$

2. subject to the constraints

$$3x_1 + 4x_2 + 3x_3 \leq 3600$$

$$2x_1 + x_2 + 3x_3 \leq 2400$$

$$x_1 + 3x_2 + 3x_3 \leq 4800$$

3. $x_1, x_2, x_3 \geq 0.$

Linear programming problem:

Introduction

Linear programming problem involving two decision variable can easily by solving by Graphical method.

Graphical solution method:

The major steps in the solution of a linear programming problem by graphical method are summarised as follows

Step 1:

Identify the problem. The decision variables objective function and the restrictions.

Step 2:

Setup the mathematical function of the problem.

Step 3:

plot a graph representing all the constrain of the problem and identify the region (solution space). The feasible region is the intersection of all the regions represented by the constrains of the problem and is restricted to the first quadrant only.

Step 4:

The feasible region obtained in step 3 may be bounded or unbounded. Compute the co-ordinates of all the corner points of the feasible region.

9) Step 5:

Find out the value of the objective function at each corner (solution) point determined in step 4.

Step 6:

Select the corner point that optimizes (maximizes or minimizes) the value of the objective function. It gives the optimum feasible solution.

2m **General Linear Programming Problem:**

We shall now consider the LPP in the general context, that is, when the number of variables is more than two.

Definition 1 (General LPP)

a) Let z be a linear function on R^n defined by

$$z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

where c_j's are constants. Let (a_{ij}) be a $m \times n$ real matrix and let $\{b_1, b_2 \dots b_m\}$ be a set of constants, such that

b)
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \geq \text{ or } \leq \text{ or } = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \geq \text{ or } \leq \text{ or } = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \geq \text{ or } \leq \text{ or } = b_m \end{cases}$$

and finally let

c) $x_j \geq 0 \quad (j = 1, 2 \ldots n)$

The problem of determining an n-tuple $(x_1, x_2 \ldots x_n)$ which makes z a minimum or maximum and which satisfies (b) and (c) is called the general Linear programming problem.

Objective function:

The linear function $z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$.

which is to be minimized (or maximized) is called the objective function of the general LPP

Constraints:

The inequations (b) are called the constraints of the general LPP.

Non-negative restrictions:

The set of inequations (c) is usually known as the set of non-negative restrictions of the general LPP.

Definition 2 (solution)

An n-tuple $(x_1, x_2 \dots x_n)$ of real numbers which Satisfied. The constrains of a general LPP is called a solution to the general LPP.

Definition 3 [feasible solution)

Any solution to the general LPP which also Satisfies the non-negative restrictions of the problem is called a feasible solution to the general LPP.

Definition 4 (optimum solution)

Any feasible solution which optimizes (minimizes or maximizes) the objective function of a general LPP is called an optimum solution to the general LPP.

Note:

The term optimal solution is also used for optimum solution.