Slack and Surplus Variables :-

Definition : 1 (Slack Variables).

Let the constrains of a General LPP be $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ $\quad ai = 1, 2 \ldots k$

Then, the non-negative variables x_{n+i} which satisfy

$$\sum_{j=1}^{n} a_{ij} + x_{n+i} = b_i \qquad i = 1, 2 \ldots k$$

are called slack variables.

Definition : 2 (surplus variables)

Let the constrains of a general LPP be $\sum_{j=1}^{n} a_{ij} x_j \geq b_i$ $\quad i = k+1, k+2 \ldots l$

Then the non-negative variables x_{n+i} which satisfy

$$\sum_{j=1}^{n} a_{ij} x_j - x_{n+i} = b_i \qquad i = k+1, k+2 \ldots l$$

3.5 Canonical and standard forms of LPP

After the formulation of linear programming problem (LPP) the next step is to obtain its solution. But for the solution of any linear programming problem, the problem must be available in a particular forms. Two forms are dealt with here, the Canonical forms and the standard forms.

The Canonical forms:

(13) The general formulation of Linear programming problem discussed in the previous section can always be put in the following forms

Maximize $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

Subject to the constraints:

$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \le b_i$;

$x_1, x_2, \dots, x_n \ge 0$

by making use of some elementary transformations. This form of LPP is called the canonical form of LPP.

(i) The objective function is of the maximization type.

The minimization of a function $f(x)$ is equivalent to the maximization of the negative expression of this function $f(x)$.

(i.e) minimize $f(x) = -$ maximize $\{-f(x)\}$.

The Standard form:

The general LPP is in the form

maximize or minimize $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

subject to the constraints

$a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n = b_i \quad i = 1, 2 \dots n$

$x_1, x_2 \dots x_n \ge 0$

is known as the standard forms

Graphical Solution Method :-

1\. Find the maximum value of $z = 4x_1 + 3x_2$ subject to constraints

14

$$2x_1 + x_2 \le 1000$$

$$x_1 + x_2 \le 800$$

$$x_1 \le 400$$

$$x_2 \le 700$$

$$x_1 \ge 0 \quad \text{and} \quad x_2 \ge 0.$$

Soln:

The given constraints are converted inequality into equality.

$$2x_1 + x_2 = 1000 \rightarrow (1)$$

$$x_1 + x_2 = 800 \rightarrow (2)$$

$$x_1 = 400 \rightarrow (3)$$

$$x_2 = 700 \rightarrow (4)$$

Put $x_1 = 0$ in (1)

$$x_2 = 1000$$

The point $A = (x_1, x_2) = (0, 1000)$.

Put $x_2 = 0$ in (1)

$$2x_1 = 1000$$

$$x_1 = \frac{1000}{2}$$

$$x_1 = 500$$

The point $B = (x_1, x_2)$

$= (500, 0)$.

Put $x_1 = 0$ in (2)

$$x_2 = 800$$

The point $C = (0, 800)$

put $x_2 = 0$ in ②

$x_1 = 800$

The point D = (800, 0)

15 The point E = (400, 0)

The point F = (0, 700)

$\therefore$ A = (0, 1000)

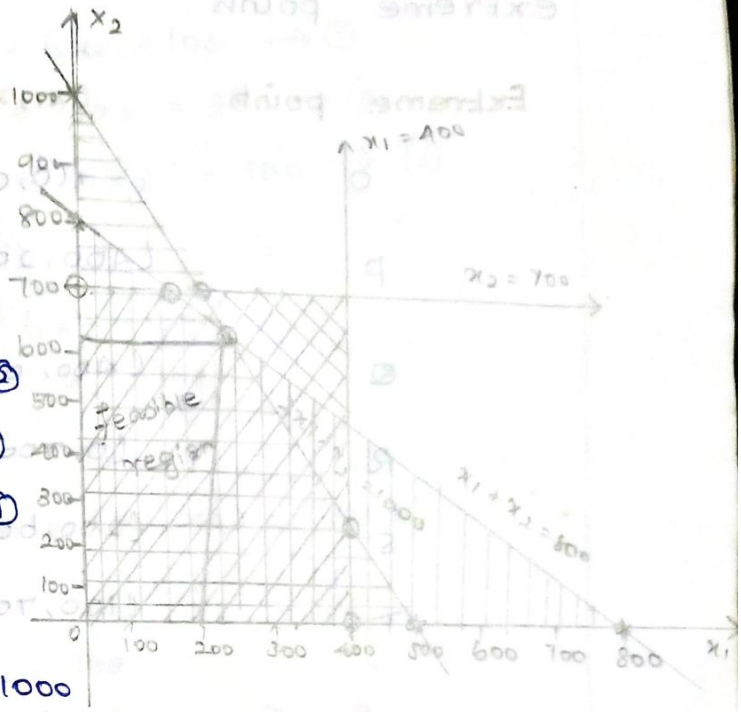
B = (500, 0)

C = (0, 800)

D = (800, 0)

E = (400, 0)

F = (0, 700)



$2x_1 + x_2 = 1000 \rightarrow$ ①

$x_1 + x_2 = 800 \rightarrow$ ②

$x_1 = 400 \rightarrow$ ③

$x_2 = 700 \rightarrow$ ④

① $\Rightarrow$ $2x_1 + x_2 = 1000$

② $\Rightarrow$ $x_1 + x_2 = 800$

$x_1 = 200$

① $\Rightarrow$ $2(200) + x_2 = 1000$

$x_2 = 1000 - 400$

$x_2 = 600$

$\therefore$ The point G = (200, 600)

put $x_1 = 400$ in ①,

$2x_1 + x_2 = 1000$

$2(400) + x_2 = 1000$

$x_2 = 1000 - 800$

$x_2 = 200$

The point H = (400, 200)

Put $x_2 = 700$ in ②

$$x_1 + x_2 = 800$$

$$x_1 = 800 - 700$$

$$x_1 = 100$$

(16)

The point $I = (100, 700)$

The point $J = (400, 0)$

The point $k = (0, 700)$

Compute the z values to the correspondin extreme points.

Extreme points	(x_1, x_2)	$z = 4x_1 + 3x_2$
O	(0,0)	0
P	(400, 200)	$z = 4(400) + 3(200)$ $= 2200$
Q	(400, 0)	1600
R	(0, 700)	2100
S	(200, 600)	2600 *
T	(100, 700)	2500

∴ The optimum solution is that extreme point for which the objective function has the largest value.

Thus, the optimum solution occurs at the point $x_1 = 200$ & $x_2 = 600$ with the objective function value of Rs. 2600.

all the constraints are of the '$\leq$' type.

1. Use graphical method to solve the following LPP :

Maximize $z = 6x_1 + x_2$

subject to the constraints

$2x_1 + x_2 \geq 3$

$x_2 - x_1 \geq 0$

$x_1, x_2 \geq 0.$

$2x_1 + x_2 = 3 \rightarrow (1)$

$x_2 - x_1 = 0 \rightarrow (2)$

put $x_1 = 0$ in (1)

$x_2 = 3.$

The point $A(x_1, x_2) = (0, 3)$

put $x_2 = 0$ in (1)

$x_1 = 3/2$

$\therefore$ The point $B = (3/2, 0).$

(i.e) $B(1.5, 0).$

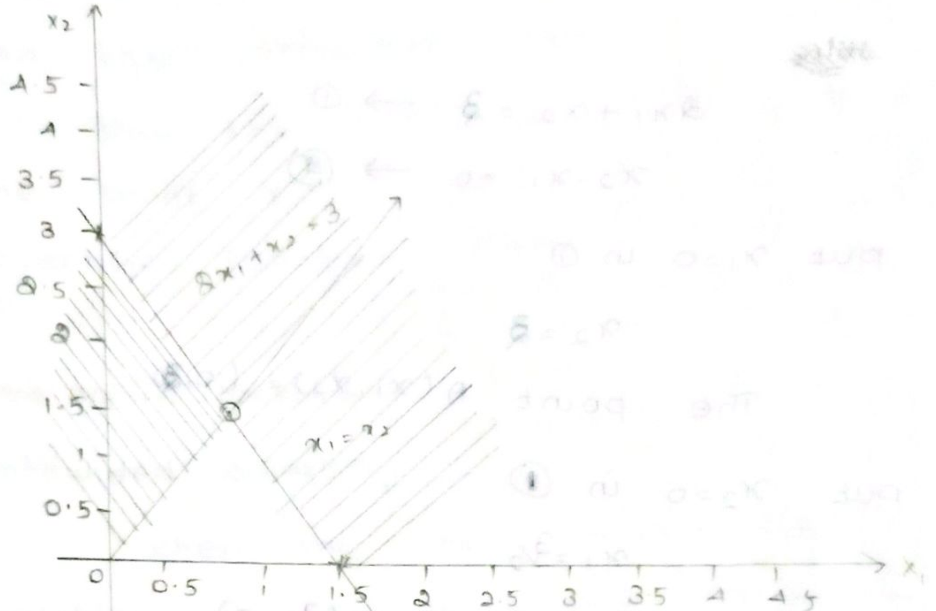
Soln:

The problem is depicted graphically the feasible the two extreme points of the region are A and B. The feasible region (solution space) is unbounded.

The value of the objective function at the extreme point $A(1,1)$ and $B(0,3)$ are 7 and 3 respectively.

But there exist number of point in the feasible region for which the value of the objective function is more than 7, thus both the variables x_1 & x_2 can be made arbitarily large and the value of Z also increased. Hence the problem has unbounded solution.

② $\Rightarrow$ $x_2 = x_1$.



2. Solve the following LPP

Maximize $Z = x_1 + x_2$.

Subject to the constraints:

$$x_1 + x_2 \leq 1$$

$$-3x_1 + x_2 \geq 3$$

$$\therefore x_1 \geq 0 \ , \ x_2 \geq 0$$

Soln:

$$x_1 + x_2 = 1 \longrightarrow ①$$

$$-3x_1 + x_2 = 3 \longrightarrow ②$$

put $x_1 = 0$ in ①

$$x_2 = 1$$

$\therefore$ The point $A = (0,1)$.

put $x_2=0$ in ①

$x_1=1$

$B=(1,0)$

put $x_1=0$ in ②

$x_2=3$

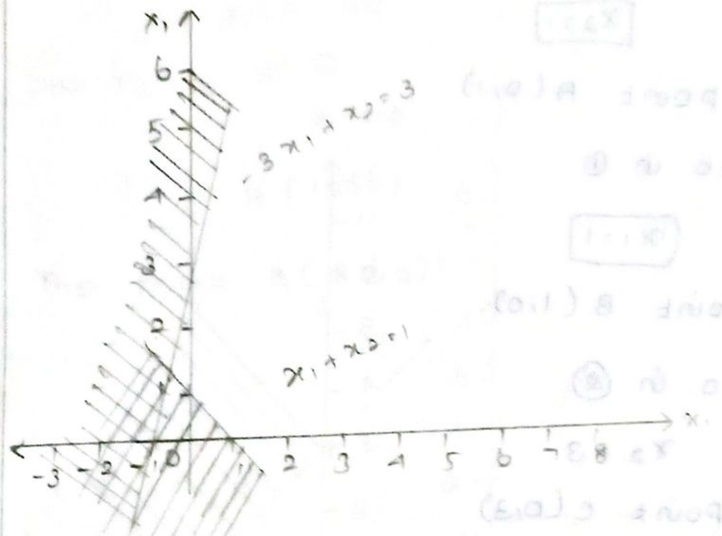
$\therefore$ The point $C(0,3)$

put $x_2=0$ in ②

$-3x_1=3$

$x_1=-1$

$\therefore$ The point $D(-1,0)$



The problem is depicted graphically in figure. As shown in the figure there is no point (x_1,x_2) which can lie in both the regions (satisfy both the constraints) there exist no solution to the given problem.

Hence there is infeasible solution.