Simplex Method :-

The simplex method also called the simplex technique or the simplex algorithm is an iterative procedure for solving a linear programming problem in a finite number of steps.

There are three types:-

- Simplex method
- Big (M) method (or) penalty method.
- Two phase method

Simplex method:

The simplex algorithm:

For the solution of LPP by simplex algorithm the existence of an initial basic feasible solution is always assumed

The steps for the computation of an optimum solution are as follows:

Step 1:

Check whether the objective function of given LPP is to maximized or minimized. If it is to be minimized then we convert into a problem of maximizing it by using the result

Maximize z = - maximize $(-z)$

Step 2:

Check whether all b_i $(i=1,2 \ldots m)$ are non-negative. If any one of b_i is negative then multiply the corresponding inequations of the constraints by -1, so as to get by all b_i $(i=1,2 \ldots m)$ non-negative.

Step 3:

Convert all the inequation of the constraints into equation by introducing slack and/or surplus variables in the constraints. put the cosets of these variables equal to zero.

Step 4:

Obtain an initial basic feasible solution to the problem in the form $X_B = B^{-1}b$ and put it in the first column of the simple table.

Step 5:

Compute the net evaluations $z_j - c_j$ ($j = 1, 2, \dots n$) by using the relation $z_j - c_j = c_B \, y_j - c_j$

Examine the sign $z_j - c_j$

(i) If all $(z_j - c_j) \geq 0$ then the initial basic feasible solution X_B is an optimum basic feasible solution.

(ii) If atleast one $(z_j - c_j) < 0$, proceed onto the next step

Step 6:

If there are more than one negative $z_j - c_j$, then choose the most negative of them. Let it be $z_r - c_r$ for some $j = r$

(i) If all $y_{ir} \leq 0$ $(i = 1, 2 \dots m)$ then there is an unbounded solution to the given problem.

(ii) If atleast one $y_{ir} > 0$ $(i = 1, 2 \dots m)$ then the corresponding vector y_r enters the basis y_B.

Step 7:

Compute the ratios

$$\left\{ \frac{x_{Bi}}{y_{ir}}, \; y_{ir} > 0, \; i = 1, 2 \dots m \right\}$$ and choose the minimum of them. Let the minimum of the ratios be x_{Br} / y_{kr}. Then the vector y_k will leave the basic y_B. The common element y_{kr}, which is in the k^{th} row and r^{th} column is known as the leading element [or pivotal element] of the table.

Step 8 :

Convert the leading element to unity by dividing its row by the leading element itself and all other element in its column to zero by making use of the relation

$$\hat{y_{ij}} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir}, \quad j = 1, 2, \ldots m+n; \quad i \neq k$$

and

$$\hat{y_{kj}} = \frac{y_{kj}}{y_{kr}} \quad j = 0, 1, 2, \ldots n$$

Step 9 :

Go to step 5 and repeat the computational procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

1. Use simplex method to solve the following LPP. $Max(z) = 4x_1 + 10x_2$.

subject to constraints:

$$2x_1 + x_2 \leq 50$$

$$2x_1 + 5x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 90$$

$$x_1, x_2 \geq 0.$$

Soln:

By introducing slack variables $x_3, x_4, x_5 \geq 0$ respectively.

The given set of constraints are converted into system of equations.

$$2x_1 + x_2 + x_3 = 50$$

$$2x_1 + 5x_2 + x_4 = 100$$

$$2x_1 + 3x_2 + x_5 = 90$$

To obtain an initial basic feasible solution

$x_3 = 50$

$x_4 = 100$

$x_5 = 90$

The general form of objective function is

$z^* = 4x_1 + 10x_2 + 0 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5$ → surplus variable

			4	10	0	0	0	
C_B (coeff of x)	Y_B (slack)	X_B (value)	Y_1	Y_2	Y_3	Y_4	Y_5	X_{Bi}/y_{ir}
0	Y_3	50	2	1	1	0	0	50/1 = 50
0	Y_4	100	2	5	0	1	0	100/5 = 20 least +ve value
0	Y_5	90	2	3	0	0	1	90/3 = 30
$z_j - c_j \geq 0$		0	-4	-10 longest -ve	0	0	0	

5 is pivotal element

Y_4 is leaving variables

Y_2 is entering variables.

cost of

			4	10	0	0	0	
C_B	Y_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	x_{iB}/y_{ir}
0	y_3	30	8/5	0	1	-1/5	0	
10	y_2	20	2/5	1	0	1/5	0	
0	y_5	30	4/5	0	0	-3/5	1	
$z_j - c_j \geq 0$		200	0	0	0	2	0	

Old $R_1 - 1$ (New R_2)

$50 - 1(20) = 30$

$2 - 1(2/5) = 8/5$

$1 - 1(1) = 0$

$1 - 1(0) = 1$

$0 - 1(1/5) = -1/5$

$0 - 1(0) = 0$

Old $R_3 - 3$ (New R_2)

$90 - 3(20) = 30$

$2 - 3(2/5) = 4/5$

$3 - 3(1) = 0$

$0 - 3(0) = 0$

$0 - 3(1/5) = -3/5$

$1 - 3(0) = 1$

Since all $Z_j - C_j \ge 0$ an optimum basic feasible solution has been reached. Hence a optimum basic feasible solution to the given LPP is

$x_1 = 0$

$x_2 = 20$

maximize $z = 800$

Check:

$2x_1 + x_2 + x_3 = 50$ $\quad\quad$ $0 + 20 + 30 = 50$

$2x_1 + 5x_2 + x_4 = 100$ $\quad\quad$ $0 + 100 + 0 = 100$

$2x_1 + 3x_2 + x_5 = 90.$ $\quad\quad$ $0 + 60 + 30 = 90$

Big (M) method : (Method of Penalties).

The Big (M) method is an alternative method of solving a linear programming problem involving artificial variables. In this method we assign a very high penalty (say M) to the artificial variable in the objective function.

The iterative procedure of the algorithm:

Step 1 :

Write the given LPP into its Standard form and check whether there exists, a starting basic feasible solution

a) If there is a ready starting basic feasible solution move on to step 4.

b) If there does not exist a ready starting basic feasible solution move on to step 2.

Step 2 :

Add artificial variables to the left side of each equation that has no obvious starting basic variables. Assign a very high penalty (say M) to these variables in the objective function.

Step 3:

Apply simplex method to the modified LPP following cases may arise at the last iteration.

a) Affect one artificial variables is present in the basic with zero value. In such a case the current optimum basic feasible solution is degenerated.

b) Atleast one artificial variable is present in the basic with a positive value. In such a case the given LPP does not possess an optimum basic feasible solution. The given problem is said to have a pseudo-optimum basic feasible solution.

1. $\max(z) = 6x_1 + 4x_2$

Subject to constraints

$2x_1 + 3x_2 \le 30$

$3x_1 + 2x_2 \le 24$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

Soln:

Introducing slack and surplus variables with an artificial variable x_3, x_4, x_5, x_6 in the constraints of given LPP.

The given set of constraints are converted into system of equations

$$2x_1 + 3x_2 + x_3 = 30$$
$$3x_1 + 2x_2 + x_4 = 24$$
$$x_1 + x_2 - x_5 + x_6 = 3$$

The general form of objective function is

$$z^* = 6x_1 + 4x_2 + 0x_3 + 0x_4 + 0x_5 - Mx_6$$

			6	4	0	0	0	-M	
CB	YB	XB	y_1	y_2	y_3	y_4	y_5	y_6	x_{iB}/y_{ir}
0	y_3	30	2	3	1	0	0	0	15
0	y_4	24	3	2	0	1	0	0	8
-M	y_6	3	1	1	0	0	-1	1	3
$z_j - c_j \ge 0$		-3M	-M-6	-M-4	0	0	0	-M	

1 is pivotal element

y_6 is leaving variable

y_1 is entering variable

			6	4	0	0	0	
CB	YB	XB	Y_1	Y_2	Y_3	Y_4	Y_5	x_B/y_{ir}
0	y_3	24	0	1	1	0	2	12
0	y_4	15	0	-1	0	1	3	5
6	y_1	3	1	1	0	0	-1	-3
$z_j - c_j \geq 0$		18	+0	+2	0	0	-6	

Old R_1 - 2(New R_3)

$30 - 2(3) = 24$

$2 - 2(1) = 0$

$3 - 2(1) = 1$

$1 - 2(0) = 1$

$0 - 2(0) = 0$

$0 - 2(-1) = 2$

Old R_2 - 3(New R_3)

$24 - 3(3) = 15$

$3 - 3(1) = 0$

$2 - 3(1) = -1$

$0 - 3(0) = 0$

$1 - 3(0) = 1$

$0 - 3(-1) = 3$

3 is pivotal element

Y_4 is leaving variable

Y_5 is entering variable

			6	4	0	0	0	
CB	YB	XB	Y_1	Y_2	Y_3	Y_4	Y_5	x_B/y_{ir}
0	Y_3	14	0	5/3	1	2/3	0	
0	Y_5	5	0	-1/3	0	1/3	1	
6	Y_1	8	1	2/3	0	1/3	6	
$z_j - c_j \geq 0$		48	0	0	0	2	0	

Since all $z_j - c_j \geq 0$ an optimum basic feasible solution has been reached.

Hence an optimum basic feasible solution to the given LPP is

$x_1 = 8$ $\quad x_4 = 0$

$x_2 = 0$ $\quad x_5 = 5$

$x_3 = 14$

max (z) = 48