Invertible matrices.

Definition:

Let A be an n×n matrix over the field F. An n×n matrix B such that BA = I is called a left inverse of A.

$$A^{-1} = \frac{\text{adj } A}{|A| \neq 0}$$

Definition:

Let A be an n×n matric over the field F. An n×n matric B such that AB = I is called a right inverse of A and A is said to be invertible

Lemma:

If A has a left inverse B and a right inverse C then B = C

proof:

Suppose BA = I and AC = I then

B = BI = B(AC) = (AB)C = IC = C.

Remark:

By the above lemma if A has left and right inverse A is invertible and has a unique to sided image inverse which are denote by A^{-1} and call the inverse of A.

Theorem : 10

Let A and B be $n \times n$ matrices over F

(i) If A is invertible. so is A^{-1} and $(A^{-1})^{-1} = A$.

(ii) If both A and B are invertible, so is AB and $(AB)^{-1} = B^{-1}A^{-1}$.

Proof:-

(i) If A is invertible. $AA^{-1} = A^{-1}A = I$

Now by symmetrary of this eqn $A^{-1} = A$

$\therefore A^{-1}A = I$

(ii) A^{-1} is invertible $(A^{-1})^{-1} = A$

(ii) we both A and B are invertible

$$AA^{-1} = A^{-1}A = I = BB^{-1} = B^{-1}B = I$$

$$\text{Now } (AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$$

$$= AIA^{-1} = AA^{-1} = I$$

(iii) If $(BB^{-1})(AB) = B^{-1}(A^{-1}A)B = BIB = B^{-1}B = I$

AB is invertible and $(AB)^{-1} = B^{-1}A^{-1}$.