Unit - I

Forces.

Linear momentum.

If m is the mass of the particle and $\bar{v}$ its velocity, then $m\bar{v}$ is called the linear momentum or simply momentum of the particle.

Units of forces

In the MKS, CGS and FPS systems the units of forces are respectively a newton, a dyne and a poundal.

Newton

A Newton is that force which, acting on a particle of mass 1kg, produces on it an acceleration of $1 m/sec^2$.

Types of forces

(i) Earth gravitation
(ii) Tension
(iii) Reaction
(iv) Friction.

Tension

Tension is a force which comes into play when an elastic body is deformed by application of forces.

Hooke's law

It has been found by experiments that tension of an elastic string or a spiral spring varies as the ratio of the extension of the string beyond its natural length to its natural length.

This fact was discovered by Hookes and hence this law is known as Hookes law.

$$\text{tension} = \lambda \frac{\text{extension}}{\text{Natural length}}.$$

Bookwork

To find the magnitude and direction of the resultant F_1 and F_2.

Solution

Now the resultant is $\bar{F}_1 + \bar{F}_2$. Let the angle between $\bar{F}_1$ and $\bar{F}_2$ be α.

Then the magnitude of $\bar{F}_1 + \bar{F}_2$ is

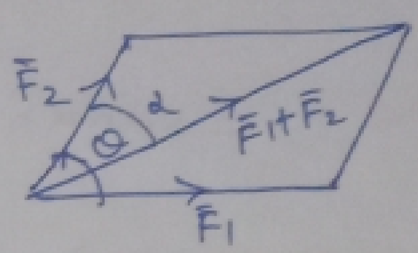
$$|F_1 + F_2| = \sqrt{(\bar{F}_1 + \bar{F}_2)(\bar{F}_1 + \bar{F}_2)}$$

$$= \sqrt{\bar{F}_1 \cdot \bar{F}_1 + \bar{F}_2 \cdot \bar{F}_2 + 2\bar{F}_1 \cdot \bar{F}_2}$$

$$|F_1 + F_2| = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\alpha}$$

Let θ be the angle between $\bar{F}_1$ and the resultant $\bar{F}_1 + \bar{F}_2$. Then

$$\tan\theta = \frac{|\bar{F}_1 \times (\bar{F}_1 + \bar{F}_2)|}{\bar{F}_1(\bar{F}_1 + \bar{F}_2)}$$



$$= \frac{|0 + \bar{F}_1 \times \bar{F}_2|}{(F_1^2 + F_1F_2\cos\alpha)}$$

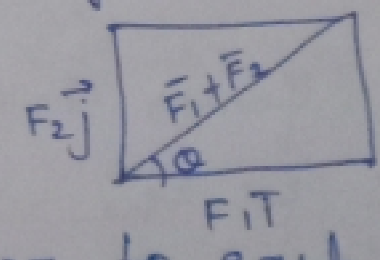
$$= \frac{|F_1F_2\sin\alpha\,\hat{n}|}{F_1(F_1 + F_2\cos\alpha)}$$

$$\tan\theta = \frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha}.$$

$$\theta = \tan^{-1}\left(\frac{F_2\sin\alpha}{F_1 + F_2\cos\alpha}\right)$$

cor. 1.

If $\bar{F}_1$ and $\bar{F}_2$ are of equal magnitude say F then $|F_1 + F_2| = 2F\cos\alpha/2$.



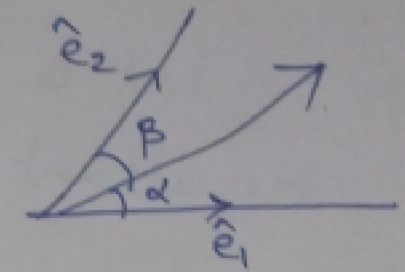
cor. 2.

If $\bar{F}_1$ and $\bar{F}_2$ are perpendicular to each other then $\tan\theta = \frac{F_2}{F_1}$ then $\theta = \tan^{-1}\frac{F_2}{F_1}$

Book work

To resolve a force $\bar{F}$ into components in two given directions.

Let $\hat{e}_1, \hat{e}_2$ be the unit vectors in the given directions. Let them makes angles α, β with $\bar{F}$.



$$\bar{F} = a\hat{e}_1 + b\hat{e}_2 \longrightarrow (1)$$

$e_1 \times (1) \Rightarrow$

$$\hat{e}_1 \times \bar{F} = a\hat{e}_1 \times \hat{e}_1 + b\hat{e}_1 \times \hat{e}_2.$$

$$F \sin\alpha \, \hat{n} = 0 + b\sin(\alpha+\beta)\hat{n}$$

$$\Rightarrow b = \frac{F\sin\alpha}{\sin(\alpha+\beta)}.$$

$$e_2 \times (1) \Rightarrow \hat{e}_2 \times \bar{F} = a\hat{e}_2 \times \hat{e}_1 + b\hat{e}_2 \times \hat{e}_2$$

$$F\sin\beta(-\hat{n}) = a\sin(\alpha+\beta)(-\hat{n})$$

$$a = \frac{F\sin\beta}{\sin(\alpha+\beta)}$$

$$\bar{F} = \frac{F\sin\beta}{\sin(\alpha+\beta)}\hat{e}_1 + \frac{F\sin\alpha\,\hat{e}_2}{\sin(\alpha+\beta)}$$

Problem

The magnitude of the resultant of the forces $\bar{F}_1$ and $\bar{F}_2$ acting on a particle is equal to the magnitude of $\bar{F}_1$. When the first force is doubled. show that the new resultant is perpendicular to $\bar{F}_2$.

Solution

Since the magnitude of $\bar{F}_1+\bar{F}_2$ is equal to the magnitude of $\bar{F}_1$.

$$|\bar{F}_1+\bar{F}_2| = |\bar{F}_1|$$

$$|\bar{F}_1+\bar{F}_2|^2 = |\bar{F}_1|^2$$

$$(\bar{F}_1+\bar{F}_2)\cdot(\bar{F}_1+\bar{F}_2) = \bar{F}_1\cdot\bar{F}_1$$

$$\bar{F}_1\cdot\bar{F}_1 + 2\bar{F}_1\cdot\bar{F}_2 + \bar{F}_2\cdot\bar{F}_2 = \bar{F}_1\cdot\bar{F}_1$$

$$2\bar{F}_1\cdot\bar{F}_2 + \bar{F}_2\cdot\bar{F}_2 = 0$$

$$(2\bar{F}_1+\bar{F}_2)\cdot\bar{F}_2 = 0.$$

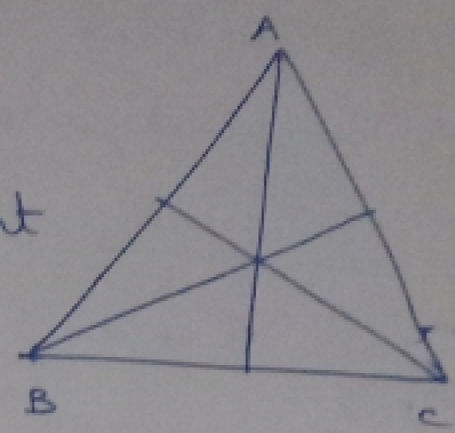
So the resultant $2\bar{F}_1$ and $\bar{F}_2$ is perpendicular to $\bar{F}_2$.

Equilibrium

When the resultant of the forces acting at a point is zero, then the forces are said to be an equilibrium

Problem

1. The sides BC, CA, AB of a ΔABC are bisected in D, E, F. Show that the forces represented by DA, EB, FC are in equilibrium.



Solution

The forces act through the centroid, since D is the midpoint of BC.

$\overline{AD} = \frac{1}{2}(\overline{AB} + \overline{AC})$

$= \frac{1}{2}(\overline{AB} - \overline{CA})$

But $\overline{DA} = -\overline{AD}$

$\overline{DA} = -\frac{1}{2}(\overline{AB} - \overline{CA})$

$\overline{EB} = -\frac{1}{2}(\overline{BC} - \overline{AB})$

$\overline{FC} = -\frac{1}{2}(\overline{CA} - \overline{BC})$

Adding these three, we get the resultant as $\overline{0}$. So the forces are in equilibrium.

2. Find the magnitude and direction of the resultant of three coplanar forces P, 2P, 3P acting at a point and inclined mutually at an angle ais of 120°.