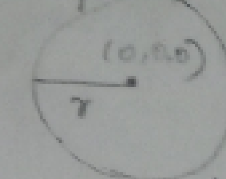
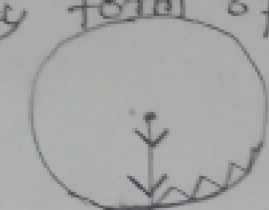
2) If the Particle is with in the sphere and not touching it's surface. The inequality holds and the particle moves freely with 3 degrees of freedom.



on the otherhand, if the Particle moves along the sphere during some interval the equality appears and the particle moves freely on the 2-dimensional surface.

③ Prove that unilateral (or) inequality form of Constraint is holomomic in nature.



Finally, if the particle hits the sphere and rebounds in accordance with some co-efficients of restitutions, the instant of impact Provides the boundary points between two periods of free motion. The initial condition of a given period being calculated from the final condition of the Previous Period.

In any event the motion of the particle is obtained by considering a sequence of holonomic system where switch occurs whenever the particle hits the spherical surface (or) leaves it. Thus the unilateral (or) inequality form of constraint is holonomic in nature.

Virtual Work :- (Imaginary)

Virtual displacement :-

Let us consider a system of N Particles whose Configuration is given by 3N Cartesian co-ordinates $x_1, x_2, \ldots, x_{3N}$.

Suppose that at any given time. Let us assume that the co-ordinates move through infinitesimal displacements $\delta x_1, \delta x_2, \ldots \delta x_{3N}$ which are virtual (imaginary) in the sense that they assumed to occur without passage of time and do not nece conform to the constrains.

This small change $\delta \vec{x}$ in the configurat the system is known as a virtual displacem

For example,

Suppose the system is subject to k- holo constrains.

$$\phi_j(x_1, x_2, \ldots, x_{3N}, t) = 0, \quad j = 1, 2, \ldots k.$$

The total differential of ϕ_j

$$d\phi_j = \frac{\partial \phi_j}{\partial x_1} dx_1 + \frac{\partial \phi_j}{\partial x_2} dx_2 + \cdots + \frac{\partial \phi_j}{\partial x_{3N}} dx_{3N} + \frac{\partial \phi}{\partial t} \quad (2)$$

$j = 1, 2, \ldots$

A virtual displacement which conform to the constraint to has the δx's related by the equat

$$\frac{\partial \phi_j}{\partial x_1} \delta x_1 + \frac{\partial \phi_j}{\partial x_2} \delta x_2 + \cdots + \frac{\partial \phi_j}{\partial x_{3N}} \delta x_{3N} = 0 \longrightarrow (3)$$

$j = 1, 2, \ldots k$

Here we have replaced dx in eqn (2) by δx and hence hold omitted the dt term because the tim is hold fixed during a virtual displacement. Similarly a system has the non- holonomic const is in the form,

Ex:-

$$\sum_{i=1}^{3N} a_{ji} dx_i + a_{jt}\, dt = 0 \longrightarrow (4)$$

$j = 1, 2, \ldots m.$

The system has m- non-holonomic constraints. virtual displacement which confirms to these constraint's must have the δx's related by the m equations.

$$\sum_{i=1}^{3N} a_{ji} \delta x_i = 0 \longrightarrow (5)$$

$j = 1, 2, \ldots m.$

(5) 2M Note:

under what condition δx can be replaced b

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The comparision of equation (2) and (3) shows that any holonomic constraint - Holonomic constraint must also be scleronomic.

(i.e) the constrain $\frac{\partial \phi_j^{\circ}}{\partial t} = 0 \longrightarrow (6)$ $j = 1, 2 \cdots k$

must apply. similarly any non-holonomic constrain must meet the condition $a_{jt} = 0 \longrightarrow (7)$ $j = 1, 2, \cdots m$

since these condition are not met in the general case. It is clear that a virtual displacement is not in general a possible real displacement.

Definition:-

Assume that the set of δx's conforming to the instantaneous occur during in the interval δt. The corresponding ratio's to of the form $\delta x / \delta t$ have the dimensions of velocity. Hence the dimension of velocity and are known as virtual velocity.

In general virtual velocities are not-possible for the actual system. It is only when (6) and (7) apply that a virtual velocity consistent with the constraint is also a possible velocity.

Definition :- virtual work :- δw

Consider a system of N-particles whose configuration is given by the cartesian co-ordinates are given by $x_1, x_2, \cdots x_{3N}$. Suppose that the force components $F_1, F_2, \cdots F_{3N}$ are applied at the corresponding coordinates in the positive sense.

The virtual work δw of these forces in a virtual displacement δx is given by $\left(\delta w = \sum_{j=1}^{3N} F_j \cdot \delta x_j\right)$

An alternate form of the expression for the virtual work $\delta w = \sum_{j=1}^{3N} \vec{F_j} \cdot \vec{\delta r_j}$. where $\vec{F_j}$ is the force applied at the j^{th} particle and where $\vec{r_j}$ is the position vector of this particle.

Note:

From the vector formulation that the virtual does not depends upon the use of any particular co-ordinate system and the motion is measured to an inertial reference frame. In the expression the virtual work the forces are assumed to remain constant throughout the virtual displacement.

The virtual work expression are defined to be lin in the virtual displacement

$$\vec{F_i}(\delta\vec{r_1}+\delta\vec{r_2}) = F_i\,\delta\vec{r_1} + F_i\,\delta\vec{r_2}$$

The virtual work is similar to a first variatio

Defn:- First variation:.

If consider a function $f(q_1, q_2, \dots, q_n)$ Continuous throughout the second partiete ai derivative the first variation of f at the refere point $\vec{q_0}$ is $\delta f = \sum_{i=1}^{n} (\partial f/\partial q_i)\, \delta q_i$,

where the δq's are the variation in the individ q's and can be consider as virtual displacement

Now consider a system which is subject to constraints let the total force acting on the i^{th} par be seperated into an applied force $\vec{F_i}$ and constra force $\vec{R_i}$.

The virtual work of the constrained force is

$$\delta w = \sum_{i=1}^{n} \vec{R_i}\cdot\delta\vec{r_i}.$$

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Definition: Workless constraints:-

A workless constraints is any bilateral constraint such that the virtual work of the