...ion then there are degre...

corresponding constraint forces is zero. For any virtual displacement which is consistent with the constraints.

For a system having only workless constraint the virtual work $\delta W_c \neq 0$ is equal to zero. or

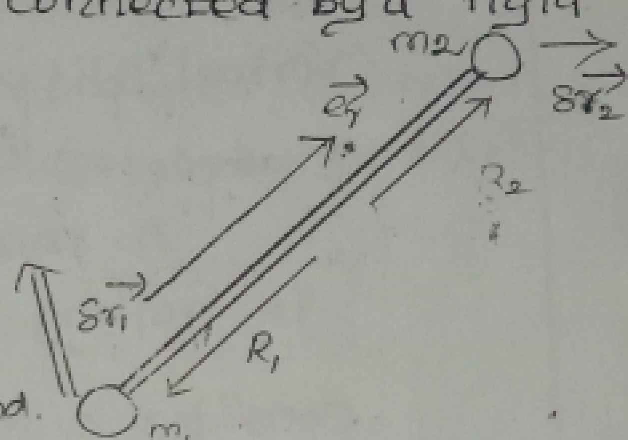
(i.e), $\sum_{i=1}^{n} \vec{R_i} \cdot \delta\vec{r_i} = 0$, where the virtual displacement $\delta\vec{r_i}$ are consistent with the instantaneous constraints.

Example of workless constraints:.

Example - 1

Prove that the constraints in rigid rod interconnection between particle is workless constraints.

Assume that two particles are connected by a rigid massless rod as in the figure.



By Newton's third law the forces exerted by the rod on the particle m_1 and m_2 are equal, opposite and collinear.

Hence, $\vec{R_2} = R_2\vec{e_r} = -\vec{R_1} = -R_1\vec{e_r}$ where $\vec{e_r}$ is the unit vector directed along the rod as shown in the figure. Furthermore since the rod is rigid the displacement component of the particle in the direction of the rod must be equal

(i.e) $\vec{e_r} \cdot \delta\vec{r_1} = \vec{e_r} \cdot \delta\vec{r_2}$

Now the virtual work of the Constraint $\delta W_c = \sum_{i=1}^{n} \vec{R_i} \cdot \delta\vec{r_i}$

$$\delta W_c = \vec{R_1} \cdot \delta\vec{r_1} + \vec{R_2} \cdot \delta\vec{r_2}$$

$$= -R_2\vec{e_r} \cdot \delta\vec{r_1} + R_2\vec{e_r} \cdot \delta\vec{r_2}$$

$$= 0$$

Therefore the virtual work of the constrained force is zero. The constraints in the rigid rod is

Bookwork :- (~~~) 8M * T 5M.

State and prove principles of virtual work :-

Statement :-

The necessary and sufficient condition for the static equilibrium of an initially motionless scleronomic system which is subject to workless constraints is that zero virtual work be done by the applied force is moving through an arbitrary virtual displacement satisfying the constraint.

Proof : Necessary condition :-

Consider a scleronomic system of N particle in static equilibrium. then for any ith particle we have $\vec{F_i}+\vec{R_i}=\vec{0}$, where $\vec{F_i}$ and $\vec{R_i}$ are applied force and constraint force of ith particle respectively.

∴ The virtual work done by all forces in moving through an arbitrary virtual displacement consistent with the constrained is zero.

$$(i.e),\quad \vec{F_i}+\vec{R_i}=0 \longrightarrow (1)$$

$$\Rightarrow (\vec{F_i}+\vec{R_i})\cdot\delta\vec{r_i}=0$$

$$\sum_{i=1}^{N}(\vec{F_i}+\vec{R_i})\cdot\delta\vec{r_i}=0$$

$$\sum_{i=1}^{N}\vec{F_i}\cdot\delta\vec{r_i}+\sum_{i=1}^{N}\vec{R_i}\cdot\delta\vec{r_i}=0 \longrightarrow (2)$$

If we assume that all the constraints are workless and if the $\delta\vec{r_i}$ is reversible (or) reversable virtual displacement consistent with the constraints,

Then
$$\sum_{i=1}^{N}\vec{R_i}\cdot\delta\vec{r_i}=0 \longrightarrow (3)$$

$$\therefore \text{Eqn (2) and (3)} \Rightarrow \sum_{i=1}^{N}\vec{F_i}\cdot\delta\vec{r_i}=0$$

$$\Rightarrow \delta W=0$$

If a system of particle with workless constraints is static equilibrium. Then it follows that the virtual work of the applied forces is zero for any

virtual displacement consistent with the con
sufficient condition :

Now assume that the same system of par
it is not in equilibrium

$$(i.e)\ \sum_{i=1}^{N} \vec{F_i} \cdot \delta\vec{r_i} = 0.$$

when the system is not in equilibrium . the virtual displacement in the direction of the motion at each point. In this case the virtual is positive

$$(i.e)\ \sum_{i=1}^{N} \vec{F_i} \cdot \delta\vec{r_i} + \sum_{i=1}^{N} \vec{R_i} \cdot \delta\vec{r_i} = 0$$

But the constraints are workless

$$\therefore \sum_{i=1}^{N} \vec{R_i} \cdot \delta\vec{r_i} = 0$$

$$\Rightarrow \delta w > 0$$

The reversed of $\delta\vec{r}$ would yield a negative vi
work of this system

But in any event if the system is not in equ
it is always possible to find a set of virtua
displacement consistent with the constraints
will result in the virtual work of the applic
forces being non-zero.

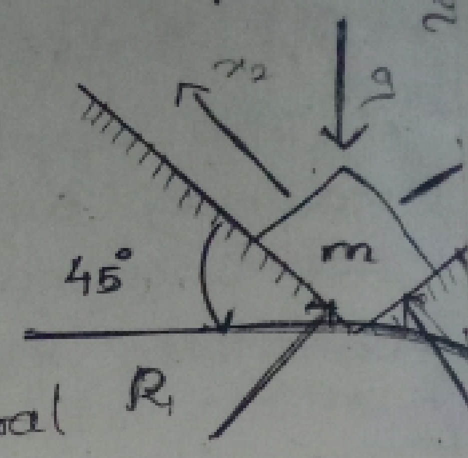
Application of principles of virtual work:

Two frictionless blocks of equal mass connected by a massless rigid rod using x_1 a
as co-ordinates, solve for the force F_2 if system is in static equilibrium (the syste
as shown in figure)

1. This is scleronomic system with work
constrains.

2. The external constrained forces are ex
floor reactions R_1 and R_2 acting pe

at a corner formed by two frictionless. mu perpenticular planes as shown in the following



Assume that any motion is restricted to the vertical plane.

Soln:

This is an example of unilatoral constrain Now the external A system with unila constrained forces of real axis R_1 and R_2 constra. the planes Ref Rerpendicular to the plane as shown figure. The only applied forces acting on this syste the gravitational force mg acting vertically d -words on the cube.

Let x_1 and x_2 be the distance measured along two planes then the unilateral constrained e are $x_1 \geq 0$, $x_2 \geq 0$. The components of mg alo direction x_1 and x_2 are $F_2 = F_1 = -mg\cos 45° = -\frac{1}{\sqrt{2}}$

$\therefore$ The virtual work of the applied force is,

$$\delta W = \sum_{i=1}^{N} \vec{F_i}\, \delta \vec{x_i}$$

$$\delta W = F_1\,\delta x_1 + F_2\,\delta x_2 = -\frac{1}{\sqrt{2}} mg\,\delta x_1 - \frac{1}{\sqrt{2}} mg$$

$$\delta W = \frac{-mg}{\sqrt{2}}(\delta x_1 + \delta x_2)$$

(i.e) The virtual work done $\delta W \leq 0$. For any vir displacement consistent of the unilateral consta

In general for an initially motionless system containing frictionless fixed constraints which may be unilateral.

The necessary and sufficient condition for static equilibrium is that the virtual work of the applied force is equal (or) less than zero. (i.e) $\delta w \leq 0$ for all virtual displacement consistent with constraint.

To calculate the virtual work of the constraint force:-

For, since the total virtual work of the forces

$(R_1+F_1)\,\delta x_1 + (R_2+F_2)\cdot\delta x_2 = 0$ (by static equilibrium)

$\Rightarrow \left(R_1 - \frac{1}{\sqrt{2}} mg\right)\delta x_1 + \left(R_2 - \frac{1}{\sqrt{2}} mg\right)\delta x_2 = 0$

Assume that δx's are not constraint.

$\Rightarrow$ the co-efficient of δx's are zero.

$\delta w = \sum_{i=1}^{N} F_i\, \delta x_i$

$= \sum_{i=1}^{N} (F_i + R_i)$

$\delta x_i = 0$

$$\Rightarrow R_1 = R_2 = \frac{1}{\sqrt{2}} mg.$$

$\therefore$ the virtual work of the constrained force R_1 and R_2 equal to

$$\delta w_c = R_1\cdot\delta x_1 + R_2\cdot\delta x_2$$

$$= \frac{1}{\sqrt{2}} mg\,\delta x_1 + \frac{1}{\sqrt{2}} mg\,\delta x_2$$

$$= \frac{1}{\sqrt{2}} mg\,(\delta x_1 + \delta x_2) = 0$$

Hence the unilateral constraints cannot be classed as workless constraint eventhough they may be frictionless.

Note:-

In the study of unilateral constraint, one finds that the constrained forces can change suddenly as the constrained functions reaches (or) leaves it's limiting value.

A similar sudden change can occur in coloumb friction forces but in this case the force is considered as a discontinued function of the sliding velocity.