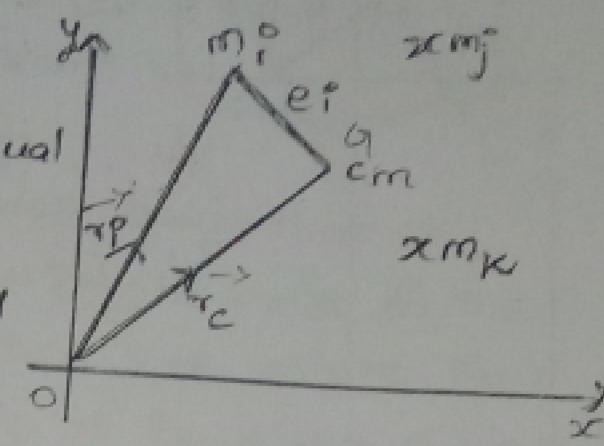
Kinetic Energy of a System of Particle.

Discuss K.E of a system of particle (or) State and prove Konig's theorem

10 mk / 5m

Statement:-

The total kinetic energy of a system is equal to sum of



i) The kinetic energy due to a particle having a mass equal to the total mass of the system and moving with velocity of the centre of mass and

ii) The K.E due to the motion of the system relative to its centre of mass.

Proof:-

Consider a system of N-particles. Let $\vec{r_i}$ be the position vector of i^{th} particle with respect to the orgin o. Fixed with an inertial frame. The total K.E of a system with respect to the inertial frame = The sum of the individual K.E of the system of particles

$$T = \sum_{i=1}^{N} \frac{1}{2} m_i r_i^2$$

$$= \sum_{i=1}^{N} \frac{1}{2} m_i (\dot{\vec{r_i}})^2$$

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i \dot{\vec{r_i}}^2 \longrightarrow (1)$$

where m_i is the mass of the i^{th} particle

Let G be the position of the centre of the mass and its position vector $\vec{r_c}$ with respect to the origin 'o'

Let $\vec{\rho_i}$ be the position of the i^{th} with respect to the centre of mass
from the diagram

$$\vec{r_i} = \vec{r_c} + \vec{\rho_i}$$

$$\therefore \dot{\vec{r_i}} = \dot{\vec{r_c}} + \dot{\vec{\rho_i}} \rightarrow (2)$$

$$\therefore \text{Equ } (1) \Rightarrow T = \frac{1}{2}\sum_{i=1}^{N} m_i \left(\dot{\vec{r_c}} + \dot{\vec{\rho_i}}\right)^2 \quad (a+b)$$

$$= \frac{1}{2}\sum_{i=1}^{N} m_i \left(\dot{\vec{r_c}} + \dot{\vec{\rho_i}}\right) \cdot \left(\dot{\vec{r_c}} + \dot{\vec{\rho_i}}\right)$$

$$T = \frac{1}{2}\sum_{i=1}^{N} m_i \left\{\dot{\vec{r_c}}^2 + 2\dot{\vec{r_c}} \cdot \dot{\vec{\rho}} + \dot{\vec{\rho}}\right.$$

$$= \frac{1}{2}\sum_{i=1}^{N} m_i \dot{\vec{r_c}}^2 + \sum_{i=1}^{N} m_i (\dot{\vec{r_c}}$$

$$+ \frac{1}{2}\sum_{i=1}^{N}$$

$$= \frac{1}{2}\dot{\vec{r_c}}^2 \sum_{i=1}^{N} m_i + \dot{\vec{r_c}} \sum_{i=1}^{N} m_i$$

$$\cdot \sum_{i=1}^{N} m_i \dot{\vec{\rho_i}}$$

$$= \frac{1}{2} m \dot{\vec{r_c}}^2 + \dot{\vec{r_c}} \cdot 0 + \frac{1}{2} \sum_{i=1}$$

$$T = \frac{1}{2} m \dot{\vec{r_c}}^2 + \frac{1}{2}\sum_{i=1}^{N} m_i \dot{\vec{\rho_i}}^2 \quad (3)$$

since ρ_i is measured from centre of m

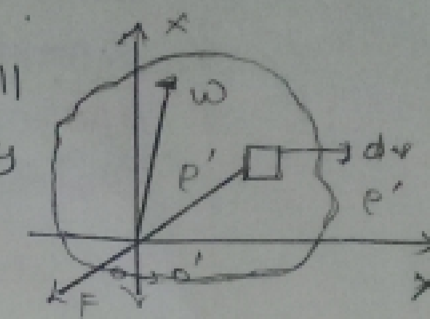
$\therefore$ Eqn (3) can be consider as the K.E of relative to its centre of mass

ie., it is the K.E of the system as vi an observer relating with centre of mass not origin

Rotational kinetic energy

K.E of a rigid body in general motion.

Suppose let us consider a small volume element dv having a density ρ'. Each element of the body when, in general we translating and rotating, the only possible exception being that an instantaneous axis of rotation might exist in the body and the elements along this line might than have no translational velocity at the given instant.



Volume element can be choosen to be so small that it's rotational K.E is negligible compared with translational K.E

Hence in the limit each element of the rigid body can be consider as a particle of infinite decimal mass. The relation in eqution (3)

$$T = \frac{1}{2} m \dot{\vec{r}}_c^{\,2} + \frac{1}{2} \sum_{i=1}^{N} m_i \dot{\rho}_i^{\,2}$$

$\frac{1}{2} \sum_{i=1}^{N} m_i \left(\dot{\vec{\rho}}_i\right)^2$ becomes $\frac{1}{2} \int_V \rho' \left(\dot{\vec{\rho}}\right)^2 dv$ — position vector with a r. to the centre of mass

where $\vec{\rho}$ is the position of the volume element with respect to the centre of mass.

$\therefore$ Total K.E of the system of rigid body motion is equal to (T),

$$T = \frac{1}{2} m \dot{\vec{r}}_c^{\,2} + \frac{1}{2} \int_V \rho' \left(\dot{\vec{\rho}}\right)^2 dv \longrightarrow (4)$$

(i.e) equal to translational K.E in the system + rotational K.E of the system.

(i.e) $T = T_{translation} + T_{rotational} \longrightarrow (5)$

(i.e) $T = T_{tran} + T_{rot}$

Let us consider the rotational K.E T_{rot} :

Take the reference point 'ρ' at the centre

Assume that the body is rotating with the velocity ω.

$\therefore$ we say that $\dot{\vec{\rho}} = \vec{\omega} \times \vec{\rho}$ ⟶ (

$\therefore \dot{\vec{\rho}}^2 = \dot{\vec{\rho}} \cdot \dot{\vec{\rho}}$

$= \vec{\omega} \times \vec{\rho} \cdot \dot{\vec{\rho}}$ (or) $\dot{\vec{\rho}} \cdot (\omega \times \vec{\rho})$

$\therefore$ The rotational K.E of the ridigid body.

$$T_{rot} = \frac{1}{2} \int_v \rho' (\dot{\vec{\rho}})^2 dv$$

$(\vec{\omega} \times \vec{\rho})$

$= [\dot{\rho}$

$= [\vec{\omega}$

$$= \frac{1}{2} \int_v \rho' (\vec{\omega} \cdot \vec{\rho} \times \dot{\vec{\rho}}) dv$$

$$= \frac{1}{2} \vec{\omega} \int_v \rho' (\vec{\rho} \times \dot{\vec{\rho}}) dv = \frac{1}{2} \vec{\omega} \int_v \rho$$

$$= \frac{1}{2} \vec{\omega} \int_v \rho' [(\vec{\rho} \cdot \vec{\rho}) \vec{\omega} - (\vec{\rho} \cdot \vec{\omega}) \cdot \vec{\rho}]$$

$$= \frac{1}{2} \int_v \rho' [(\vec{\rho} \cdot \vec{\rho}) \vec{\omega}^2 - (\vec{\rho} \cdot \vec{\omega}) (\vec{\rho}$$

$$= \frac{1}{2} \int_v \rho' (\vec{\rho})^2 (\vec{\omega})^2 - (\vec{\rho} \cdot \vec{\omega})^2] dv.$$

Let $\vec{i}, \vec{j}$ and $\vec{k}$ be the unit vector along a Ca co-ordinates system with its origin at the of mass and assume that it rotates body. we have,

$$\vec{\rho} = x\vec{i} + y\vec{j} + z\vec{k} \text{ and } \vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega$$

$$\therefore \text{Eqn (1)} \Rightarrow T_{rot} = \frac{1}{2} \int_v \rho' [(x^2+y^2+z^2)(\omega_x^2 +$$

$$- (x\omega_x + y\omega_y + z\omega_z)^2] d$$

$$= \frac{1}{2} \int_v \rho' [(x^2+y^2+z^2)\omega_x^2 + (x^2+y^2+z^2)\omega_y^2 + (x^2$$

$= \frac{1}{2}$

... description than there are ...

$$-x^2\omega_x^2 - y^2\omega_y^2 - z^2\omega_z^2 - 2xy\,\omega_x\omega_y - 2xz\,\omega_x\omega_z - 2yz\,\omega_y\omega_z]\,dv$$

$$= \frac{1}{2}\int_V \rho'(y^2+z^2)\,\omega_x^2\,dv + \frac{1}{2}\int_V \rho'(x^2+z^2)\,\omega_y^2\,dv + \frac{1}{2}\int_V \rho'(x^2+y^2)\,\omega_z^2\,dv - \int_V \rho' xy\,\omega_x\omega_y\,dv - \int_V \rho' xz\,\omega_x\omega_z\,dv - \int_V \rho' yz\,\omega_y\omega_z\,dv.$$

$$T_{rot} = \frac{1}{2}\left[\int_V \rho'(y^2+z^2)\,dv\right]\omega_x^2 + \frac{1}{2}\left[\int_V \rho'(x^2+z^2)\,dv\right]\omega_y^2 + \frac{1}{2}\left[\int_V \rho'(x^2+y^2)\,dv\right]\omega_z^2 - \left(\int_V \rho' xy\,dv\right)\omega_x\omega_y - \left(\int_V \rho' xz\right)\omega_x\omega_z - \left(\int_V \rho' yz\,dv\right)\omega_y\omega_z$$

$$T_{rot} = \frac{1}{2} I_{xx}\,\omega_x^2 + \frac{1}{2} I_{yy}\,\omega_y^2 + \frac{1}{2} I_{zz}\,\omega_z^2 - [(I_{xy}\,\omega_x\omega_y + I_{xz}\,\omega_x\omega_z + I_{yz}\,\omega_y\omega_z)] \quad \text{--- (8)}$$

where

$$\left.\begin{aligned} I_{xx} &= \int_V \rho'(y^2+z^2)\,dv \\ I_{yy} &= \int_V \rho'(x^2+z^2)\,dv \\ I_{zz} &= \int_V \rho'(x^2+y^2)\,dv \end{aligned}\right\}$$ are the moment of inertia.

$$\left.\begin{aligned} I_{xy} = I_{yx} &= -\int_V \rho' xy\,dv \\ I_{yz} = I_{zy} &= -\int_V \rho' yz\,dv \\ I_{zx} = I_{xz} &= -\int_V \rho' zx\,dv \end{aligned}\right\}$$ are the product of inertia.

Equation (8) can be written as

$$T_{rot} = \frac{1}{2}(\omega_x\ \omega_y\ \omega_z)\begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix}\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

$T_{rot} = \frac{1}{2} \vec{\omega}^T \cdot I \vec{\omega}$ //

If $\vec{\omega}$ has the same direction has one of the axis and a given moment (say x-axis)

$\Rightarrow \vec{\omega_z} = \vec{\omega_y} = 0$

$\therefore$ In this case, $T_{rot} = \frac{1}{2} I_{xx} \omega_x^2$ (or) T_{rot}

where I is the moment inertia about the

Note:

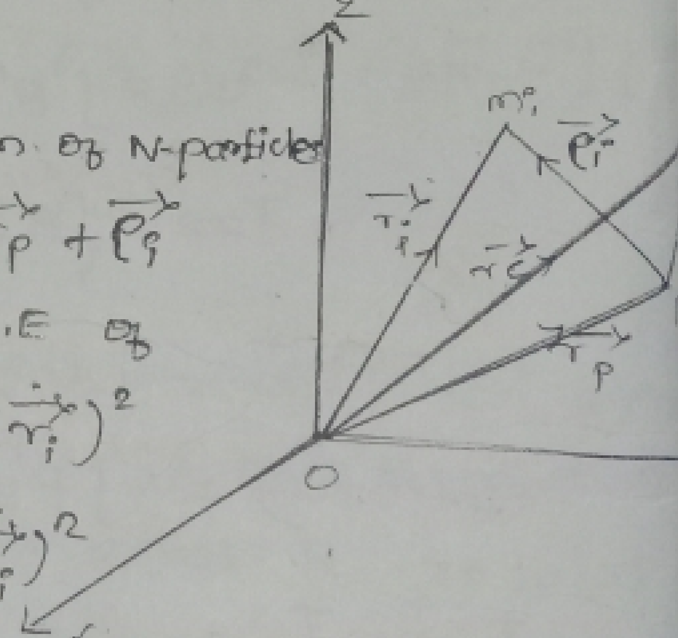
In above two expression for the K.E of system & terms of its motion relation to fixed point and also in terms of motion to its centre of mass.

5m Book work :-

Find an expression for the K.E of the sys terms of its motion with respect to an point

Soln:-

consider a system of N-particles



from the diagram $\vec{r_i} = \vec{r_p} + \vec{\rho_i}$

We know that, total K.E of N particle $T = \frac{1}{2} \sum_{i=1}^{N} m_i (\dot{\vec{r_i}})^2$

Now,

$$T = \frac{1}{2} \sum_{i=1}^{N} m_i (\dot{\vec{r_p}} + \dot{\vec{\rho_i}})^2$$

$$= \frac{1}{2} \sum_{i=1}^{N} m_i (\dot{\vec{r_p}} + \dot{\vec{\rho_i}}) \cdot (\dot{\vec{r_p}} + \dot{\vec{\rho_i}})$$

$$= \frac{1}{2} \sum m_i (\dot{\vec{r_p}})^2 + \frac{1}{\cancel{2}} \times \cancel{2} \sum_{i=1}^{N} m_i (\dot{\vec{r_p}} \cdot$$

$$+ \frac{1}{2} \sum_{i=1}^{N} m_i$$

$$= \frac{1}{2} (\dot{\vec{r_p}})^2 \sum_{i=1}^{N} m_i + \dot{\vec{r_p}} \sum_{i=1}^{N} m_i \dot{\vec{\rho_i}} + \frac{1}{2}$$

$$+ \frac{1}{2} \sum_{i=1}^{N} m_i ($$

this description then there are ...

Since centre of mass location ρ_c at p. Therefore

$$m\vec{\rho}_c = \sum_{i=1}^{N} m_i \vec{\rho}_i \text{ and } \sum_{i=1}^{N} m_i = m \text{ and hence}$$

$$m\dot{\vec{\rho}}_c = \sum_{i=1}^{N} \dot{\vec{\rho}}_i \cdot m_i$$

$$\boxed{T = \frac{1}{2} m \dot{\vec{r}}_p^{\,2} + \dot{\vec{r}}_p \cdot m\dot{\vec{\rho}}_c + \frac{1}{2}\sum_{i=1}^{N} m_i (\dot{\vec{\rho}}_i)^2} \rightarrow (I)$$

Thus we find that the total K.E is the sum of three parts,

i) The K.E due to a particle having a mass 'm' and moving with the reference point p

ii) The K.E of the system due its motion relative to p and

iii) The scalar product of the velocity of the reference point and their Linear momentum of a system relative to its reference point

Note:-

when centre of mass at $p \Rightarrow \dot{\vec{\rho}}_c = 0 \Rightarrow$ I reduce to

$$(I) \Rightarrow T = \frac{1}{2} m \dot{\vec{r}}_p^{\,2} + \frac{1}{2}\sum_{i=1}^{n} m_i (\dot{\vec{\rho}}_i)^2$$

Remark:-

The K.E of a rigid body interms of its motion relative to an arbitrary reference point p

Proof:-

The K.E due to its motion relative to its p is $T_{rot} = \frac{1}{2} m \dot{\vec{\rho}}_c^{\,2} + \frac{1}{2}\sum_i \sum_j I_{ij} \vec{\omega}_i \vec{\omega}_j$ where the moments and products of inertia are taken with respect to the mass centre then the total K.E =

$$T = \frac{1}{2} m \dot{\vec{r}}_p^{\,2} + \frac{1}{2} m \dot{\vec{\rho}}_c^{\,2} + \frac{1}{2}\sum_i \sum_j I_{ij} \vec{\omega}_i \cdot \vec{\omega}_j + \dot{\vec{r}}_p \cdot m\dot{\vec{\rho}}_c$$

If it turns out that the motion of body relative to P is a pure rotation, i.e., P is a fixed in the body then the relative motion can be written in the

$$T_{rot} = \frac{1}{2}\sum_i \sum_j I_{ij}\, \vec{\omega}_i \cdot \vec{\omega}_j$$ where the

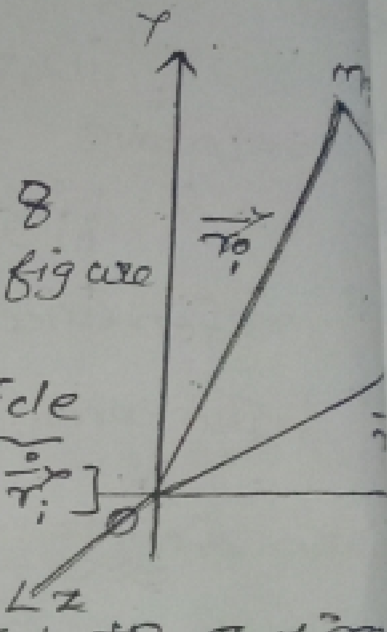
and the products of inertia are taken to a point P

(x) Angular momentum :- (AM)

10m

(ME)

i) For the case of system of particles as shown in the figure



2m

the A.M of the i^{th} particle

$$= \vec{r}_i \times m_i \dot{\vec{r}}_i = \vec{r}_i \times m_i \dot{\vec{r}}_i\,]$$

The total A.M $\vec{H}$ with respect to a fixed 'O' is

$$\vec{H} = \sum_{i=1}^{N} \vec{r}_i \times m_i \dot{\vec{r}}_i \quad \text{--- (1)}$$

i.e., it is the sum of the moments of the individual linear momenta of the par assuming that each vector $m_i \dot{\vec{r}}_i$ has a line action passing through the corresponding P

Since $\vec{r}_i = \vec{r}_c + \vec{\rho}_i$

$$\vec{A} \times \vec{B}$$

$$\therefore \dot{\vec{r}}_i = \dot{\vec{r}}_c + \dot{\vec{\rho}}_i$$

$$H = \sum_{i=1}^{N}$$

Therefore equ (1). implies

$$\vec{H} = \sum_{i=1}^{N} \vec{r}_i \times m_i \,(\dot{\vec{r}}_c + \dot{\vec{\rho}}_i)$$

$$\vec{H} = \sum_{i=1}^{N} [\vec{r}_c + \vec{\rho}_i] \times m_i (\dot{\vec{r}}_c + \dot{\vec{\rho}}_i)$$

... their description then there are ...

$$= \sum_{i=1}^{N} \vec{r}_c \times m_i \dot{\vec{r}}_c + \sum_{i=1}^{N} \vec{r}_c \times m_i \dot{\vec{\rho}}_i + \sum_{i=1}^{N} \vec{\rho}_i \times m_i \dot{\vec{r}}_c + \sum_{i=1}^{N} \vec{\rho}_i \times m_i \dot{\vec{\rho}}_i \longrightarrow (2)$$

Since, $\sum_{i=1}^{N} m_i \vec{\rho}_i = 0$ and $\sum_{i=1}^{N} m_i = m$.

Therefore equation (2) $\Rightarrow$

$$\vec{H} = \vec{r}_c \times m \dot{\vec{r}}_c + \sum_{i=1}^{N} \vec{\rho}_i \times m_i \dot{\vec{\rho}}_i \longrightarrow (3)$$

$\therefore \vec{H} =$ [The A.M of a system of particle of total mass 'm' about a fixed point 'o' is equal to the A.M about 'o' of a single particle of mass 'm' which is moving with the centre of mass] + The A.M of the system about the centre of mass

(ie) $\vec{H} = \vec{r}_c \times m \dot{\vec{r}}_c + \vec{H}_c$ where $\vec{H}_c = \sum_{i=1}^{N} \rho_i \times m_i \dot{\vec{\rho}}_i$

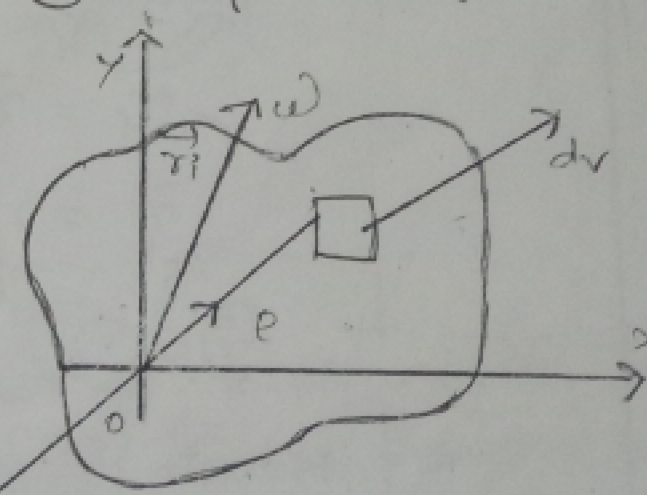
= A.M about the centre of mass.

(ii) For the case of rigid body in arbitrary motion :-

The total A.M with respect to a fixed point 'o'.

$$\vec{H} = \vec{r}_c \times m \dot{\vec{r}}_c + \vec{H}_c \quad \text{where} \quad \vec{H}_c = \int_V \rho' (\vec{\rho} \times \dot{\vec{\rho}})\, dv$$

$$\vec{H}_c = \int_V \rho' (\vec{\rho} \times \vec{\omega} \times \vec{\rho})\, dv \longrightarrow (4) \qquad \dot{\vec{\rho}} = \vec{\omega} \times \vec{\rho}$$



where ρ' is the density of the small volume element dv.

$\vec{\omega}$ is the angular velocity of the body.

In terms of moments of inertia and products of inertia about the centre of mass, we obtain the body axis components of $\vec{H}_c$ From (or) For the matric equation,

$$\vec{H}_c = I \vec{\omega}$$

Also comparing the equation

$$T_{rot} = \frac{1}{2}\vec{\omega}\cdot\int_V \rho'(\vec{\rho}\times(\vec{\omega}\times\vec{\rho}))\,dv.$$

and
$$\vec{H}_c = \int_V \rho'(\vec{\rho}\times(\vec{\omega}\times\vec{\rho}))\,dv.$$

$$\therefore T_{rot} = \frac{1}{2}\vec{\omega}\cdot\vec{H}_c$$

$$\left[\text{Also } T_{rot} = \frac{1}{2}\vec{\omega}^T I\,\vec{\omega} = \frac{1}{2}\omega^T \vec{H}_c\right]$$

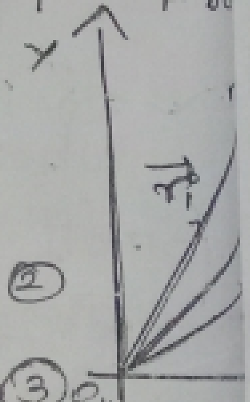
Also since thus we have obtain for the A.M system of particle with respect to the fi point and with respect to the centre of m

(iii) The A.M with respect to an arbitrary poi

The A.M about the point P, as view non-rotating observer. moving with that define

$$\vec{H}_P = \sum_{i=1}^{N} \rho_i \times m_i \dot{\vec{\rho}}_i \longrightarrow (A)$$

where $\vec{\rho}_i$ is the position of the i^{th} part with respect to the reference point P. Fro figure,



$$\vec{r}_i = \vec{r}_p + \vec{\rho}_i \Rightarrow \vec{\rho}_i = \vec{r}_i - \vec{r}_p \longrightarrow (1)$$

$$\vec{r}_c = \vec{r}_p + \vec{\rho}_c \Rightarrow \vec{r}_p = \vec{r}_c - \vec{\rho}_c \longrightarrow (2)$$

(2) in (1)

$$\Rightarrow \vec{\rho}_i = \vec{r}_i - (\vec{r}_c - \vec{\rho}_c) = \vec{r}_i - \vec{r}_c + \vec{\rho}_c \longrightarrow (3)$$ and $m\vec{r}_c =$

Now (3) in (A)

$$\dot{\vec{\rho}}_i =$$

$$\vec{H}_P = \sum_{i=1}^{N} (\vec{r}_i - \vec{r}_c + \vec{\rho}_c) \times m_i \dot{\vec{\rho}}_i$$

$$= \sum_{i=1}^{N} (\vec{r}_i - \vec{r}_c + \vec{\rho}_c) \times m_i (\dot{\vec{r}}_i - \dot{\vec{r}}_c + \dot{\vec{\rho}}_c)$$

$$= \sum_{i=1}^{N} \vec{r}_i \times m_i \dot{\vec{r}}_i - \sum_{i=1}^{N} \vec{r}_i \times m_i \dot{\vec{r}}_c + \sum_{i=1}^{N}$$

$$- \sum_{i=1}^{N} \vec{r}_c \times m_i \dot{\vec{r}}_i + \sum_{i=1}^{N} \vec{r}_c \times m_i \dot{\vec{r}}_c - \sum_{i=1}^{N} \vec{r}_c$$

their description then there

$$+\sum_{i=1}^{N}\vec{e}_c \times m_i \dot{\vec{r}}_i - \sum_{i=1}^{N}\vec{e}_c \times m_i \dot{\vec{r}}_c + \sum_{i=1}^{N}\vec{e}_c \times m_i \dot{\vec{e}}_c \longrightarrow (4)$$

consider,

$$m\vec{r}_c = \sum_{i=1}^{N} m_i \vec{r}_i \text{ and } m\dot{\vec{r}}_c = \underbrace{\sum_{i=1}^{N} m_i \dot{\vec{r}}_i}\,;\ \sum_{i=1}^{N} m_i = m$$

Now,

$$-\sum_{i=1}^{N}\vec{r}_i \times m_i \dot{\vec{r}}_c = -\sum_{i=1}^{N} m_i \vec{r}_i \times \dot{\vec{r}}_c = -m\vec{r}_c \times \dot{\vec{r}}_c$$

$$\sum_{i=1}^{N}\vec{r}_i \times m_i \dot{\vec{e}}_c = \underbrace{\sum_{i=1}^{N} m_i \vec{r}_i} \times \dot{\vec{e}}_c = m\vec{r}_c \times \dot{\vec{e}}_c$$

$$-\sum_{i=1}^{N}\vec{r}_c \times m_i \dot{\vec{r}}_i = -\vec{r}_c \times \underbrace{\sum_{i=1}^{N} m_i \dot{\vec{r}}_i} = -\vec{r}_c \times m\dot{\vec{r}}_c$$

$$\sum_{i=1}^{N}\vec{r}_c \times m_i \dot{\vec{r}}_c = \vec{r}_c \times m\dot{\vec{r}}_c \longrightarrow (5)$$

$$-\sum_{i=1}^{N}\vec{r}_c \times m_i \dot{\vec{e}}_c = -\vec{r}_c \times m\dot{\vec{e}}_c$$

$$\sum_{i=1}^{N}\vec{e}_c \times m_i \dot{\vec{r}}_i = \vec{e}_c \times \sum_{i=1}^{N} m_i \dot{\vec{r}}_i = \vec{e}_c \times m\dot{\vec{r}}_c$$

$$-\sum_{i=1}^{N}\vec{e}_c \times m_i \dot{\vec{r}}_c = -\vec{e}_c \times m\dot{\vec{r}}_c$$

$$\sum_{i=1}^{N}\vec{e}_c \times m_i \dot{\vec{e}}_c$$

equation (4) and (5) $\Rightarrow$

$$\Rightarrow H_p = \sum_{i=1}^{N}\vec{r}_i \times m_i \dot{\vec{r}}_i - \vec{r}_c \times m\dot{\vec{r}}_c + \vec{e}_c \times m\dot{\vec{e}}_c$$

(i.e) $\vec{H}_p = \vec{H} - \vec{r}_c \times m\dot{\vec{r}}_c + \vec{e}_c \times m\dot{\vec{e}}_c$ //

where $\vec{H} = \sum_{i=1}^{N}\vec{r}_i \times m_i \dot{\vec{r}}_i$

(2) **Generalized momentum :- (G.M)**

consider, a system whose configuration is described by n-generalized co-ordinates. Define a lagrangean function $L(q, \dot{q}, t)$ as $L = T - V \longrightarrow (1)$

The generalized momentum p_i associated with the generalized co-ordinate q_i is defined by the equation.

$$p_i = \partial L / \partial \dot{q}_i \longrightarrow (2)$$

It is an in general a function of q's, $\dot{q}$'s

If the potential energy is of the form,

therefore $\partial V / \partial \dot{q}_i = 0$

$$\text{since } p_i = \partial L / \partial \dot{q}_i = \partial / \partial \dot{q}_i (T - V) = \partial T / \partial \dot{q}_i$$

$$= \partial T /$$

Note: K.E − P.E.

The Lagrangean function is almost q_i, $\dot{q}$. Therefore p_i is linear functions of the $\dot{q}$

Example: 1)

Consider a free particle of mass 'm' position is given by the cartesian co-ordinates (x, y, z).

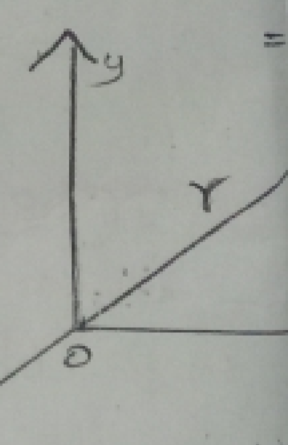
$$T = \frac{1}{2} m \dot{\vec{r}}^2$$

$$= \frac{1}{2} m (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$$

$\therefore$ The K.E is $T = \frac{1}{2} m \dot{\vec{r}}^2$

$$(\because \vec{r} = x\vec{i} + y\vec{j} + z\vec{k})$$

$$\dot{\vec{r}} = \dot{x}\vec{i} + \dot{y}\vec{j} + \dot{z}\vec{k})$$



Since the G.M about the x-axis

$$p_x = \partial T / \partial \dot{x} = m\dot{x},$$

Similarly, $p_y = m\dot{y}$ and $p_z = m\dot{z}$

The G.M about y-axis $p_y = m\dot{y}$

The G.M about z-axis $p_z = m\dot{z}$

(i.e) p_x is the x components of the linear

Example: −2