

2

Type: 1

$$1) \int \frac{5x+7}{6x^2+4x-1} dx.$$

Sol:

$$5x+7 = A \frac{d}{dx} (6x^2+4x-1) + B$$

$$5x+7 = A (12x+4) + B$$

$$5x+7 = 12Ax + 4A + B$$

Equating coeff of "x"

$$5 = 12A$$

$$A = \frac{5}{12}$$

Equating coeff of constant

$$7 = 4A + B$$

$$7 = A \left( \frac{5}{12} \right) + B$$

$$B = 7 - \frac{5}{3}$$

$$B = \frac{21-5}{3}$$

$$B = \frac{16}{3}$$

$$\text{Ans} \quad 5x+7 = \frac{5}{12} (12x+4) + \frac{16}{3}$$

$$\int \frac{5x+7}{6x^2+4x-1} dx = \int \frac{\frac{5}{12} (12x+4) + \frac{16}{3}}{6x^2+4x-1} dx$$

$$= \frac{5}{12} \int \frac{12x+4}{6x^2+4x-1} dx + \frac{16}{3} \int \frac{dx}{6x^2+4x-1}$$

$$\frac{5}{12} \log(6x^2 + 4x - 1) + \frac{16}{3} \int \frac{dx}{6\left(x^2 + \frac{4}{6}x - \frac{1}{6}\right)}$$

$$\frac{5}{12} \log(6x^2 + 4x - 1) + \frac{16}{3 \times 6} \int \frac{dx}{6\left(x^2 + \frac{4}{6}x - \frac{1}{6}\right)}$$

$$\frac{5}{12} \log(6x^2 + 4x - 1) + \frac{8}{18} \int \frac{dx}{x^2 + \frac{2}{3}x - \frac{1}{6}}$$

$$\frac{5}{12} \log(6x^2 + 4x - 1) + \frac{8}{9} \int \frac{dx}{x^2 + \frac{2}{3}x - \frac{1}{6}}$$

$$(ax^2 + bx) = \left(ax + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

$$= \left(x + \frac{2/3}{2}\right)^2 - \left(\frac{2/3}{2}\right)^2$$

$$= \left(x + \frac{2}{3} \times \frac{1}{2}\right)^2 - \left(\frac{2}{3} \times \frac{1}{2}\right)^2$$

$$= \left(x + \frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2$$

$$= \left(x + \frac{1}{3}\right)^2 - \frac{1}{9} - \frac{1}{6}$$

$$= \left(x + \frac{1}{3}\right)^2 - \frac{1 \times 2}{9 \times 2} - \frac{1 \times 3}{6 \times 3}$$

$$= \left(x + \frac{1}{3}\right)^2 - \frac{2}{18} - \frac{3}{18}$$

$$= \left(x + \frac{1}{3}\right)^2 - \frac{2+3}{18}$$

$$= \left(x + \frac{1}{3}\right)^2 - \frac{5}{18}$$



$$\frac{5}{18} \log(6x^2 + 4x - 1) + \frac{8}{9} \int \frac{dx}{\left(x + \frac{1}{3}\right)^2 - \frac{5}{18}}$$

$$\frac{5}{18} \log(6x^2 + 4x - 1) + \frac{8}{9} \int \frac{dx}{\left(x + \frac{1}{3}\right)^2 - \sqrt{\frac{5}{18}}^2}$$

$$\frac{5}{18} \log(6x^2 + 4x - 1) + \frac{8}{9} \cdot \frac{1}{2x \sqrt{\frac{5}{18}}} \log \left[ \frac{x + \frac{1}{3} - \sqrt{\frac{5}{18}}}{x + \frac{1}{3} + \sqrt{\frac{5}{18}}} \right]$$

$$\frac{5}{18} \log(6x^2 + 4x - 1) + \frac{4}{9} \sqrt{\frac{18}{5}} \log \left[ \frac{\left(x + \frac{1}{3}\right) - \sqrt{\frac{5}{18}}}{\left(x + \frac{1}{3}\right) + \sqrt{\frac{5}{18}}} \right] "$$

2)  $\int \frac{2x + 3}{x^2 + x + 1} dx.$

Sol:

$$2x + 3 = A \frac{d}{dx}(x^2 + x + 1) + B$$

$$2x + 3 = A(2x + 1) + B$$

$$2x + 3 = 2Ax + A + B$$

Equating coeff of "x"

$$2 = 2A$$

$$A = 2/2$$

$$A = 1.$$

Equating coeff of constant

$$A + B = 3$$

$$1 + B = 3$$

$$B = 3 - 1 \Rightarrow B = 2$$

$$2x+3 = 1(2x+1) + 2$$

$$\int \frac{2x+3}{x^2+x+1} = \int \frac{(2x+1) + 2}{x^2+x+1} dx$$

$$\int \frac{2x+1}{x^2+x+1} dx + 2 \int \frac{dx}{x^2+x+1}$$

$$ax + b$$

$$(ax^2 + bx) = \left(ax + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

$$= \left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + 1$$

$$= \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{4}{4}$$

$$= \left(x + \frac{1}{2}\right)^2 - \frac{1+4}{4}$$

$$= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\log(x^2+x+1) + 2 \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \sqrt{\frac{3}{4}}}$$

$$\log(x^2+x+1) + 2 \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\log(x^2+x+1) + \frac{1}{\sqrt{3/2}} \tan^{-1} \left( \frac{x+1/2}{\sqrt{3/2}} \right)$$

$$\log(x^2+x+1) + \frac{4}{\sqrt{3}} \tan^{-1} \left( \frac{2x+1}{\sqrt{3}} \right)$$

$$3) \int \frac{2x+1}{x^2+3x+1} dx$$

Sol:

$$2x+1 = A \frac{d}{dx} (x^2+3x+1) + B$$

$$2x+1 = A(2x+3) + B$$

$$2x+1 = 2Ax + 3A + B$$

Equating coeff of "x"

$$2 = 2A$$

$$A = 2/2$$

$$A = 1$$

Equating coeff of constant

$$3A + B = 1$$

$$B = 1 - 3A$$

$$B = 1 - 3(1)$$

$$B = 1 - 3$$

$$B = -2$$

$$2x+1 = A(2x+3) + B$$

$$2x+1 = 1(2x+3) + (-2)$$

$$2x+1 = 1(2x+3) - 2$$

$$\int \frac{2x+1}{x^2+3x+1} dx = \int \frac{1(2x+3) - 2}{x^2+3x+1} dx$$

$$\int \frac{2x+1}{x^2+3x+1} dx = \int \frac{2x+3}{x^2+3x+1} dx - 2 \int \frac{1}{x^2+3x+1} dx$$

$$\log(x^2+3x+1) - 2 \int \frac{1}{x^2+3x+1} dx$$



$$(ax + b)^2 = \left(ax + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

$$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + 1$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{4}{4}$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{9+4}{4}$$

$$= \left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$$

$$= \log(x^2 + 3x + 1) - 2 \int \frac{dx}{\left(x + \frac{3}{2}\right)^2 - \frac{5}{4}}$$

$$= \log(x^2 + 3x + 1) - 2 \int \frac{dx}{\left(x + \frac{3}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2}$$

$$= \log(x^2 + 3x + 1) - 2 \frac{1}{2\left(\frac{\sqrt{5}}{2}\right)} \log \left[ \frac{x + \frac{3}{2} - \frac{\sqrt{5}}{2}}{x + \frac{3}{2} + \frac{\sqrt{5}}{2}} \right]$$

$$= \log(x^2 + 3x + 1) - \frac{2}{\sqrt{5}} \log \left[ \frac{2x + 3 - \sqrt{5}}{2x + 3 + \sqrt{5}} \right] //$$

Type : 2

$$1) \int \frac{3x-4}{\sqrt{3x^2+4x+7}} dx.$$

Sol:

$$3x-4 = A \frac{d}{dx} (3x^2+4x+7) + B$$

$$3x-4 = A(6x+4) + B$$

$$3x-4 = 6xA + 4A + B$$

Equating coeff of "x"

$$6A = 3$$

$$A = \frac{3}{6}$$

$$A = \frac{1}{2}$$

Equating coeff of constant

$$4A + B = -4$$

$$-4 - 4 \left( \frac{1}{2} \right) = B$$

$$B = -4 - 2$$

$$B = -6$$

$$\int \frac{3x-4}{\sqrt{3x^2+4x+7}} dx = \int \frac{\frac{1}{2}(6x+4) - 6}{\sqrt{3x^2+4x+7}} dx$$

$$\frac{1}{2} \int \frac{6x+4}{\sqrt{3x^2+4x+7}} dx - 6 \int \frac{dx}{\sqrt{3x^2+4x+7}}$$

$$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$$

$$= \frac{1}{2} \cdot 2 \sqrt{3x^2+4x+7} - 6 \int \frac{dx}{\sqrt{3} \left( x^2 + \frac{4}{3}x + \frac{7}{3} \right)}$$



$$= \sqrt{3x^2 + 4x + 7} - \frac{6}{\sqrt{3}} \int \sqrt{x^2 + \frac{4}{3}x + \frac{7}{3}}$$

$$(ax^2 + bx) = \left(ax + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

$$= \left(x + \frac{4/3}{2}\right)^2 - \left(\frac{4/3}{2}\right)^2$$

$$= \left(x + \frac{4}{3} \times \frac{1}{2}\right)^2 - \left(\frac{4}{3} \times \frac{1}{2}\right)^2$$

$$= \left(x + \frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^2$$

$$= \left(x + \frac{2}{3}\right)^2 - \frac{4}{9} + \frac{7 \times 3}{3 \times 3}$$

$$= \left(x + \frac{2}{3}\right)^2 - \frac{4}{9} + \frac{21}{9}$$

$$= \left(x + \frac{2}{3}\right)^2 - \frac{4 + 21}{9}$$

$$= \left(x + \frac{2}{3}\right)^2 + \frac{17}{9}$$

$$= \sqrt{3x^2 + 4x + 7} - \frac{6}{\sqrt{3}} \int \frac{dx}{\left(x + \frac{2}{3}\right)^2 + \sqrt{\frac{17}{9}}}$$

$$= \sqrt{3x^2 + 4x + 7} - \frac{6}{\sqrt{3}} \int \frac{dx}{\sqrt{\left(x + \frac{2}{3}\right)^2 + \frac{\sqrt{17}}{3}}}$$

$$= \sqrt{3x^2 + 4x + 7} - \frac{6}{\sqrt{3}} \left[ \frac{x + 2/3}{\sqrt{17/3}} \right]$$

$$= \sqrt{3x^2 + 4x + 7} - \frac{6}{\sqrt{3}} \sin^{-1} \left[ \frac{3x + 2}{\sqrt{17}} \right]$$



$$2) \int \frac{2x+3}{\sqrt{4x^2-3x+3}} dx.$$

Sol:

$$2x+3 = A \frac{d}{dx} (4x^2-3x+3) + B$$

$$2x+3 = A(8x-3) + B$$

$$2x+3 = 8Ax - 3A + B$$

Equ coeff of "x"

$$2 = 8A$$

$$A = \frac{2}{8}$$

$$A = \frac{1}{4}$$

Equ coeff of constant

$$-3A + B = 3$$

$$-3\left(\frac{1}{4}\right) + B = 3$$

$$B = 3 + \frac{3}{4}$$

$$B = \frac{12+3}{4}$$

$$B = \frac{15}{4}$$

$$2x+3 = \frac{1}{4}(8x-3) + \frac{15}{4}$$

$$\frac{2x+3}{\sqrt{4x^2-3x+3}} = \int \frac{\frac{1}{4}(8x-3) + \frac{15}{4}}{\sqrt{4x^2-3x+3}} dx$$

$$\frac{1}{4} \int \frac{8x-3}{\sqrt{4x^2-3x+3}} dx + \frac{15}{4} \int \frac{dx}{\sqrt{4x^2-3x+3}}$$

$$\frac{dx}{\sqrt{x}} = 2\sqrt{x}$$

$$= \frac{1}{2} \cdot \frac{1}{2} \sqrt{4x^2 - 3x + 3} + \frac{15}{4} \int \frac{dx}{\sqrt{4\left(x^2 - \frac{3}{4}x + \frac{3}{4}\right)}}$$

$$= \frac{1}{2} \sqrt{4x^2 - 3x + 3} + \frac{15}{4 \times 2} \int \frac{dx}{\sqrt{x^2 - \frac{3}{4}x + \frac{3}{4}}}$$

$$(ax^2 + bx) = \left(ax + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

$$= \left(x - \frac{3/4}{2}\right)^2 - \left(\frac{3/4}{2}\right)^2$$

$$= \left(x - \frac{3}{4} \times \frac{1}{2}\right)^2 - \left(\frac{3}{4} \times \frac{1}{2}\right)^2$$

$$= \left(x - \frac{3}{8}\right)^2 - \left(\frac{3}{8}\right)^2$$

$$= \left(x - \frac{3}{8}\right)^2 - \frac{9}{64} + \frac{3 \times 16}{4 \times 16}$$

$$= \left(x - \frac{3}{8}\right)^2 - \frac{9}{64} + \frac{48}{64}$$

$$= \left(x - \frac{3}{8}\right)^2 + \frac{39}{64}$$

$$= \frac{1}{2} \sqrt{4x^2 - 3x + 3} + \frac{15}{8} \int \frac{dx}{\sqrt{\left(x - \frac{3}{8}\right)^2 + \frac{39}{64}}}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left[ x + \sqrt{x^2 + a^2} \right] \text{ (or) } \sin^{-1} \frac{x}{a}$$

$$\frac{1}{2} \sqrt{4x^2 - 3x + 3} + \frac{15}{8} \int \frac{dx}{\sqrt{\left(x - \frac{3}{8}\right)^2 + \left(\frac{\sqrt{39}}{8}\right)^2}}$$

$$\frac{1}{2} \sqrt{4x^2 - 3x + 3} + \frac{15}{8} \log \left[ \left(x - \frac{3}{8}\right) + \sqrt{\left(x - \frac{3}{8}\right)^2 + \left(\frac{\sqrt{39}}{8}\right)^2} \right]$$



$$\frac{1}{2} \sqrt{4x^2 - 3x + 3} + \frac{15}{8} \log \left( \frac{8x-3}{8} \right) + \sqrt{\left(x - \frac{3}{8}\right)^2 + \left(\frac{\sqrt{39}}{8}\right)^2}$$

3)  $\int \sqrt{\frac{x+1}{x-1}} dx.$

sol:

$$\int \frac{\sqrt{x+1}}{\sqrt{x-1}} dx \Rightarrow \int \frac{\sqrt{x+1}}{\sqrt{x-1}} \times \sqrt{x+1} dx$$

$$= \int \frac{x+1}{\sqrt{x-1} \cdot \sqrt{x+1}} dx$$

$$= \int \frac{x+1}{\sqrt{(x-1)(x+1)}} dx$$

$$= \int \frac{x+1}{\sqrt{x^2+x-x-1}} dx$$

$$= \int \frac{x+1}{\sqrt{x^2-1}} dx$$

$$x+1 = A \left( \frac{d}{dx} (x^2-1) \right) + B$$

$$x+1 = A(2x) + B$$

$$x+1 = 2Ax + B$$

Equ coeff of "x"

$$1 = 2A \Rightarrow A = \frac{1}{2}$$



Equ coeff of constants

$$B = 1.$$

$$x+1 = \frac{1}{2}(2x) + 1$$

$$\int \frac{x+1}{\sqrt{x^2-1}} dx \Rightarrow \int \frac{\frac{1}{2}(2x) + 1}{\sqrt{x^2-1}} dx$$

$$= \frac{1}{2} \int \frac{2x}{\sqrt{x^2-1}} + 1 \int \frac{dx}{\sqrt{x^2-1}}$$

$$dx/\sqrt{x} = 2\sqrt{x}$$

$$= \frac{1}{2} \cdot 2 \sqrt{x^2-1} + \int \frac{dx}{\sqrt{x^2-1}}$$

$$\left[ \int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) \right] \Rightarrow \text{Formula}$$

$$= \sqrt{x^2-1} + \cosh^{-1}\left(\frac{x}{1}\right)$$

$$= \sqrt{x^2-1} + \cosh^{-1}(x) //$$