3) $\dfrac{dx}{1+\sin x+\cos x}$

Sol:

$$dx = 2dt/_{1+t^2} \quad \sin x = 2t/_{1+t^2} \quad \cos x = 1-t^2/_{1+t^2}$$

$$= \int \frac{2dt/1+t^2}{1+\left(\frac{2t}{1+t^2}\right)+\left(\frac{1-t^2}{1+t^2}\right)}$$

$$= \int \frac{2dt/1+t^2}{\frac{1+t+2t+1-t^2}{\cancel{1+t^2}}}$$

$$= 2\int \frac{dt}{2+t}$$

$$= \frac{2}{2}\int \frac{dt}{t+1}$$

$$= \int \frac{dt}{t+1}$$

$$= \log\,(t+1)$$

$$= \log\left[1+\tan x/2\right]$$

Hence, $\log\left[1+\tan x/2\right]$ //.

Type : 6

1) $\displaystyle\int \frac{2\sin x + 3\cos x + 3}{5+8\cos x}\, dx.$

Sol:

$$2\sin x + \cos x + 3 = A(5+8\cos x) + B(-8\sin x) + C$$

Equating $\sin x$ and $\cos x$

$8A = 1$ $\quad$ $-8B = 2$ $\quad$ $5A + C = 3$

$A = 1/8$ $\quad$ $B = -1/4$ $\quad$ $C = 3 - 5/8$

$C = 19/8$

$$2\sin x + \cos x + 3 = \tfrac{1}{2}(5+8\cos x) - \tfrac{1}{4}(-8\sin x) + 19/8.$$

$$\int \frac{2\sin x + \cos x + 3}{5+8\cos x}\, dx = \int \frac{\frac{1}{8}(5+8\cos x) \div \frac{1}{4}(-8\sin x) + C}{5+8\cos x}$$

$$= \tfrac{1}{8}\int dx - \tfrac{1}{4}\int \frac{-8\sin x}{5+8\cos x}\, dx + \tfrac{19}{8}\int \frac{dx}{5+8\cos x}$$

$$= \tfrac{1}{8}x - \tfrac{1}{4}\log(5+8\cos x) + \tfrac{19}{8}\int \frac{dx}{5+8\cos x}$$

To evaluate,

$$\int \frac{dx}{5+8\cos x}$$

$$\int \frac{dx}{5+8\cos x} = \int \frac{2dt/1+t^2}{5+8\left(\frac{1-t^2}{1+t^2}\right)}$$

$$= 2\int \frac{dt}{5(1+t^2)+8(1-t^2)}$$

$$= 2\int \frac{dt}{13-3t^2}$$

$$= 2\int \frac{dt}{3\left(13/3 - t^2\right)}$$

$$= 2/3 \int \frac{dt}{\left(\frac{\sqrt{13}}{3}\right)^2 - t^2}$$

$$\int \frac{dx}{5+8\cos x} = 2/3 \cdot \frac{1}{2\times\sqrt{\frac{13}{3}}} \log\left[\frac{\sqrt{13/3}+t}{\sqrt{13/3}-t}\right]$$

$\therefore \tan x/2$

Substituting (A). (B).

$$\int \frac{2\sin x+\cos x+3}{5+8\cos x}\,dx = 1/8\,x - 1/4 \log(5+8\cos x) + \frac{1}{\sqrt{39}}\left(\frac{\sqrt{13/3}+t}{\sqrt{13/3}-t}\right).$$

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2) $\int \frac{2\sin x+\cos x}{5\sin x-3\cos x}$

Sol:

$2\sin x+\cos x = A(5\sin x-3\cos x)+B(5\cos x+3\sin x)+C=0$

$$2\sin x + \cos x = 5A\sin x - 3A\cos x + 5B\cos x + 3B\sin x + C$$

$$5A + 3B = 2 \rightarrow (2) \times 3$$

$$-3A + 5B = 1 \rightarrow (3) \times 5$$

$$\cancel{15A} + 9B = 6$$

$$-\cancel{15A} + 25B = 5$$

$$34B = 11$$

$$B = 11/34.$$

$$5A + 3\left(11/34\right) = 2$$

$$5A = 2 - \frac{33}{4}$$

$$= \frac{68 - 33}{34}$$

$$= \frac{35}{34}$$

$$A = \frac{\cancel{35}^{7}}{34} \times \frac{1}{\cancel{5}_{1}}$$

$$A = \frac{7}{4}$$

$$C = 0$$

$$\int \frac{2\sin x + \cos x}{5\sin x - 3\cos x}\,dx = \int \frac{7/34\,(5\sin x - 3\cos x) + 11/34\,(5\cos x + 3\sin x)\,dx}{5\sin x - 3\cos x}$$

$$= \frac{7}{34}\int \frac{5\sin x - 3\cos x}{5\sin x - 3\cos x}\,dx + \frac{11}{34}\int \frac{5\cos x + 3\sin x}{5\sin x - 3\cos x}\,dx.$$

$nx+c$

$$= \frac{7}{34}(x) + \frac{11}{34}\log(5\sin x - 3\cos x)$$

3) $\displaystyle\int \frac{\sin x + 18\cos x}{3\sin x + 4\cos x}\,dx.$

Sol:

$\sin x + 18\cos x = A(3\sin x + 4\cos x) + B(3\cos x - 4\sin x) + c.$

$\sin x + 18\cos x = 3A\sin x + 4A\cos x + 3B\cos x - 4B\sin x + c$

$3A - 4B = 1 \longrightarrow ② \times 3$

$4A + 3B = 18 \longrightarrow ③ \times 4$

$$\begin{aligned} 9A - \cancel{12B} &= 3 \\ 16A + \cancel{12B} &= 72 \\ \hline 25A &= 75 \end{aligned}$$

$A = 75/25$

$A = 3$

$3(3) - 4B = 1$

$-4B = 1 - 9$

$B = -8/-4$

$B = 2$

$x+$

dx

$\sin x + 18\cos x = 3(3\sin x + 4\cos x) + 2(3\cos x - 4\sin x)$

$$\int \frac{\sin x + 18\cos x}{3\sin x + 4\cos x}\,dx = \int \frac{3(3\sin x + 4\cos x) + 2(3\cos x - 4\sin x)}{3\sin x + 4\cos x}$$

$x.$

$$= 3\int \frac{3\sin x + 4\cos x}{3\sin x + 4\cos x}\,dx + 2\int \frac{3\cos x - 4\sin x}{3\sin x + 4\cos x}\,dx$$

$$= 3(x) + 2\log(3\sin x + 4\cos x).$$