~~1.2C Defn:~~

~~As the A set which has no elements~~

1.2C Defn:

The set having no element is called the empty set or null set. It is denoted by ϕ or $\{\ \}$.

Ex: Let $A=\{1,2\}$ & $B=\{3,4\}$

We have $A\cap B=\phi$.

Remark: (i) $A\cup\phi=A$ (ii) $A\cap\phi=\phi$

(iii) The empty set is a subset of every set.

1.2D Defn:

If A and B are sets, then B−A is the set of all elements of B which are not elements of A.

That is $B-A=\{x / x\in B, x\notin A\}$.

Ex:

Let $A=\{1,2,3\}$ & $B=\{3,4,5\}$.

We have $B-A=\{4,5\}$.

1.2E Defn:

Let A and B be two sets, if every element of A also an element of B, is said to be A is contained in B, (or)

That A is a subset of B. It is denoted by "$A \subset B$."

If A is contained in B, then we also say that B contains A. It is denoted by "$B \supset A$".

Ex:

If $A = \{1, 3, 4\}$,

$B = \{1, 2, 3, 4, 5\}$ & $C = \{2, 3, 5\}$

then $A \subset B$ and

$B \supset C$.

Note: Proper subset defn.

(Page no. 10)

1.2 E Defn: Equal (Page no. 8)

Note: $A = B$ if and only if

$A \subset B$ and $B \subset A$.

1.2 G Defn: complement (Page no 13)

give to example & properties

1.2 H Theorem:

If A, B are subsets of S,

then prove that

$(A \cup B)' = A' \cap B' \longrightarrow (1)$

& $(A \cap B)' = A' \cup B' \longrightarrow (2)$

Proof:

given A, B are subsets of S.

First we have to discuss proof

of (1), $(A \cup B)' = A' \cap B'$.

ie, $(A \cup B)' \subset A' \cap B' \rightarrow (i)$ and

$A' \cap B' \subset (A \cup B)' \rightarrow (ii)$

Let $x \in (A \cup B)'$,

then $x \notin (A \cup B)$.

Thus x is an element of neither A nor B.

ie, $x \notin A$ & $x \notin B$

So that $x \in A'$ and $x \in B'$.

Thus $x \in A' \cap B'$.

Hence $(A \cup B)' \subset A' \cap B'$

It satisfies the condition (i).

Conversely,

if $y \in A' \cap B'$,

then $y \in A'$ and $y \in B'$,

So that $y \notin A$ and $y \notin B$.

Thus ~~$y \notin A$~~ y is an element of complement neither A nor B.

So that $y \notin A \cup B$, and

So $y \in (A \cup B)'$.

Hen $(A' \cap B') \subset (A \cup B)'$

It satisfies the condition (ii)

From (i) & (ii) we get

$(A \cup B)' = A' \cap B'$.

Hence proved.

Next we have to prove

That (2), $(A \cap B)' = A' \cup B'$.

ie, $(A \cap B)' \subset A' \cup B' \longrightarrow$ (iii)

and $A' \cup B' \subset (A \cap B)' \longrightarrow$ (iv)

Let $x \in (A \cap B)'$,

then $x \notin A \cap B$.

Thus $x \notin A$ and $x \notin B$,

Hence x is an element of

neither A' nor B',

So that $x \in A'$ Or $x \in B'$

ie, $x \in A' \cup B'$

Hence $(A \cap B)' \subset A' \cup B'$ it

satisfies the condition (iii).

conversely,

Let $y \in A' \cup B'$,

then $y \in A'$ or $y \in B'$,

So that $y \notin A$ or $y \notin B$.

Thus y is not an element

of A nor B.

So that $y \notin (A \cap B)$

Hence $y \in (A \cap B)'$

Therefore $A' \cup B' \subset (A \cap B)'$

it satisfies the condition (iv)

Hence from (iii) & (iv) we have,

$(A \cap B)' = A' \cup B'$

Hence proved.

1.3 Functions

1.3 B Defn:

If A, B are sets, then the Cartesian product of A and B is the set of all ordered pairs $\langle a, b \rangle$ where $a \in A$ and $b \in B$.
It is denoted by $A \times B$.

Thus the cartesian product of the set of real numbers with itself gives the set of all ordered pairs of real numbers.

Example: page no. 13

1.3 C Defn:

Let A and B be any two sets. A function f from A into B is a subset of $A \times B$ with the property that each $a \in A$ belongs to precisely one pair $\langle a, b \rangle$.

Instead of $\langle x, y \rangle \in f$, it should be write $y = f(x)$.
Then y is called the image of x under f. The set A is called the domain of f.
The range of f is the set $\{ b \in B \,/\, b = f(a) \}$.

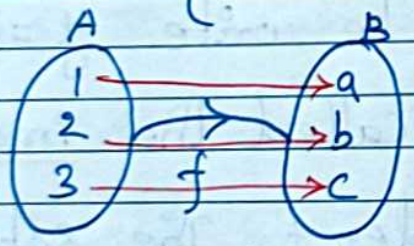
That is, the range of f is the subset of B consisting of all images of elements of A, Such a function is called a mapping of A into B.

Ex:

① If $f : A \to B$

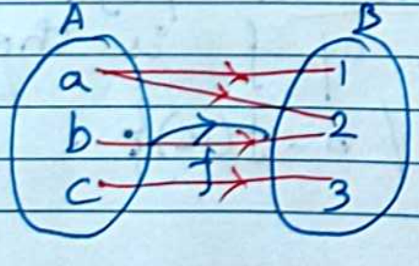
where $A = \{1, 2, 3\}$ &

$B = \{a, b, c\}$ is a function.



$\therefore f : A \to B$ is function.

②



$f : A \to B$ is not a function.

③ The set $f = \left\{ \langle x, x^2 \rangle / -\infty < x < \infty \right\}$

is the function usually described by the equation

Note:

Suppose f and g are two functions with respective domains X and Y. If $X \subset Y$ and if $f(x) = g(x)$ $(x \in X)$, We say that g is an extension of f to Y or that f is the restriction of g to X.

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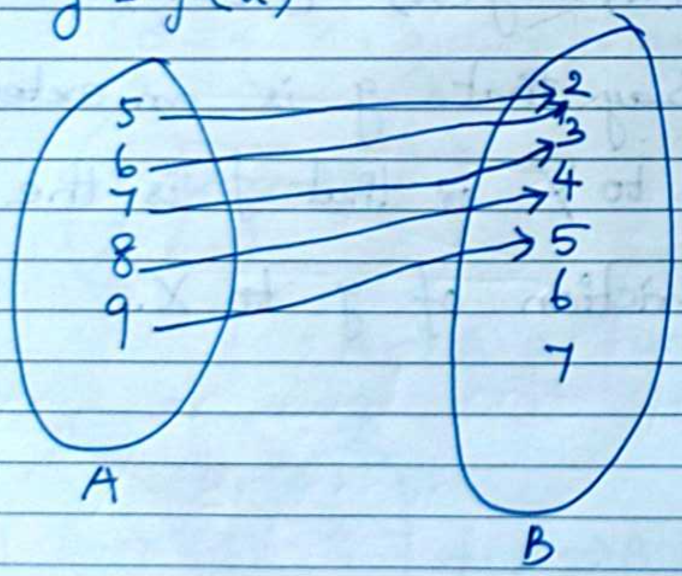
Basic Another Defn!

1) Function:

A function is a relation that map each element x of a set A with only one element y of set B.

Function $f: A \to B$ is given as $y = f(x)$

Ex:



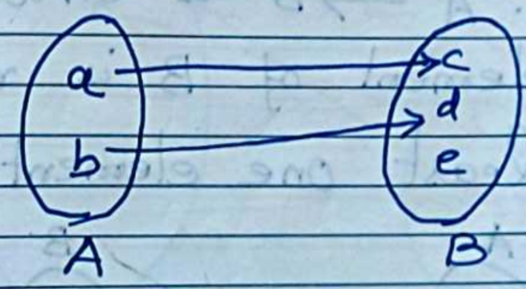
Date: Page No.: 23

Types of Functions

1) One to one / Injective Function

$f: A \to B$ is one to one if every element of A has distinct image in B.

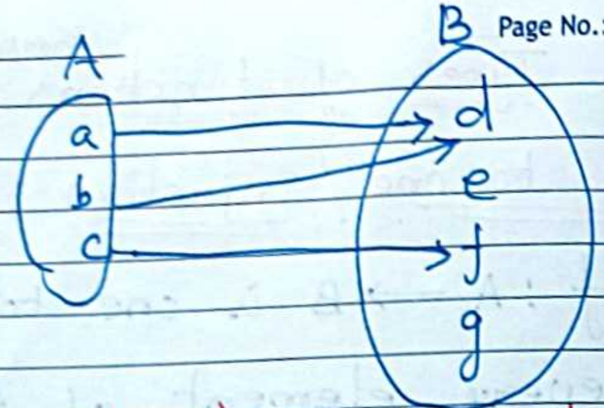
Ex:



2) Many to one function:

$f: A \to B$ is many to one if two or more elements of A have same image in B.

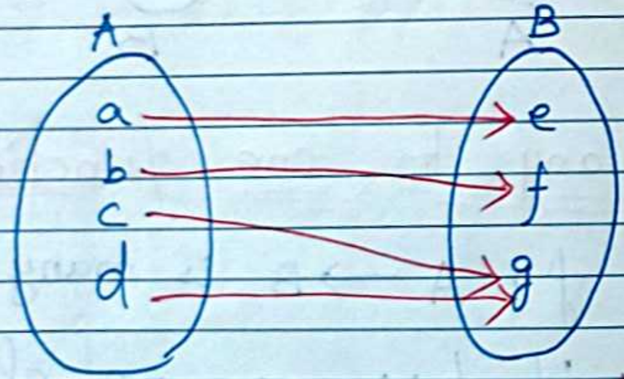
Ex:



3) Onto function/Surjective:

$f : A \to B$ is onto if every element of B is related to at least one element of A

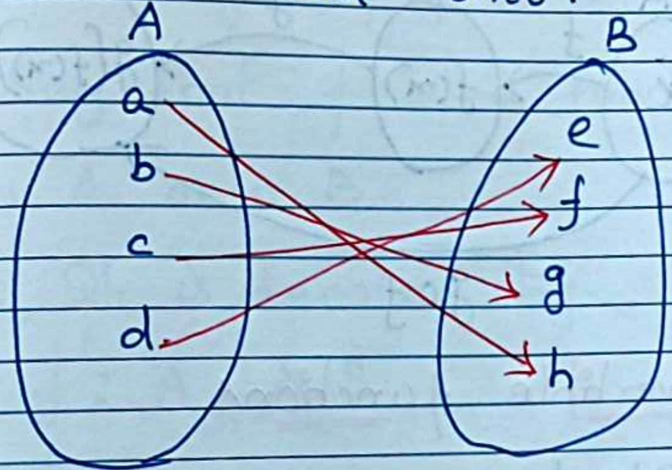
Ex:



4) One-one and onto function / Bijective

$f : A \to B$ is one-one and onto / bijective if it satisfies both the condition for one-one and onto.

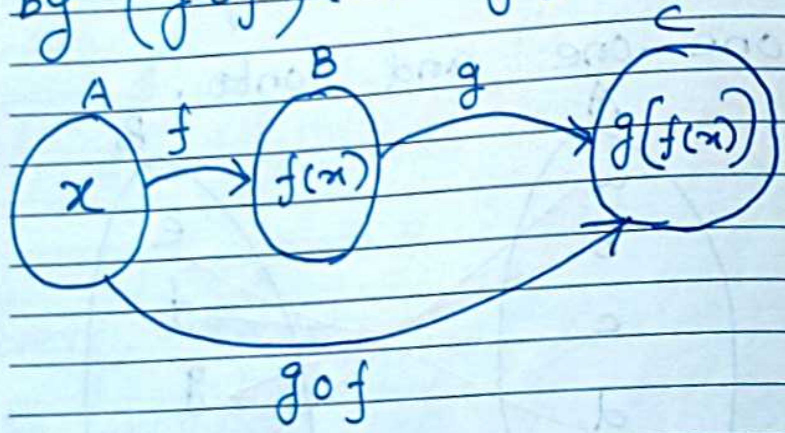
Ex:



Composite function:

$f : A \to B$ and $g : B \to C$ can be composed to form a function which maps from A to C.

A composite function ~~which~~ ~~maps from~~ A to C. is denoted by $(g \circ f)(x) = g[f(x)]$



Invertible functions

If $f : A \to B$ then

$$f^{-1} : B \to A$$

for a function to be invertible it needs to be both one-one and onto function.

1.3 D Defn:

If f is a function from A into B, That is $f : A \to B$.

If the range of f is all of B, we say that f is a function from A onto B.

ie, It is denoted by

$$f : A \Rightarrow B.$$

1.3 E Theorem!

If $f : A \to B$ and

if $X \subset B$, $Y \subset B$ then

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y).$$

i, The inverse image of the union of two sets is the union of the inverse images.

Proof: Given $f: A \to B$ and

$$X \subset B,\ Y \subset B.$$

To prove that:

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y).$$

ie, we have to prove that

$$f^{-1}(X \cup Y) \subset f^{-1}(X) \cup f^{-1}(Y) \to (1)$$

$$f^{-1}(X) \cup f^{-1}(Y) \subset f^{-1}(X \cup Y) \to (2)$$

Suppose $a \in f^{-1}(X \cup Y)$.

Then $f(a) \in X \cup Y$.

Hence either $f(a) \in X$ or $f(a) \in Y$.

So that either $a \in f^{-1}(X)$

or $a \in f^{-1}(Y)$.

But this says,

$$a \in f^{-1}(X) \cup f^{-1}(Y).$$

$$\therefore f^{-1}(X \cup Y) \subset f^{-1}(X) \cup f^{-1}(Y)$$

it satisfies the condition (1).

Conversely, if $b \in f^{-1}(X) \cup f^{-1}(Y)$, then either $b \in f^{-1}(X)$ or $b \in f^{-1}(Y)$.

Thus we have, either $f(b) \in X$ or $f(b) \in Y$.

So that $f(b) \in X \cup Y$,

then $b \in f^{-1}(X \cup Y)$

Hence,

$$f^{-1}(X) \cup f^{-1}(Y) \subset f^{-1}(X \cup Y)$$

it satisfies the condition (2).