UNIT-IV

Gaussion Elimination Method:

Let the system be

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{aligned}\right\} \longrightarrow (1)$$

we first form the augument matrix of the system (1) viz.....

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \longrightarrow (2)$$

we multiply the first equation by $-\frac{a_{21}}{a_{11}}$ and the add it to the second equation.

Similarly eliminant x_i from third equation we multiply the first equation by $-\frac{a_{31}}{a_{11}}$ and add it to the third. This procedure can be shown this

$$-\frac{a_{21}}{a_{11}} \begin{bmatrix} a_{11} & b_{11} & c_{11} & b_1 \\ a_{21} & b_{21} & c_{21} & b_2 \\ a_{31} & b_{31} & c_{31} & b_3 \end{bmatrix} \longrightarrow (3)$$

Where $-\frac{a_{21}}{a_{11}}$ and $-\frac{a_{31}}{a_{11}}$ are called the multiplication for the first stage of eliminati

In this stage, we have assumed that $a_{11} \neq 0$ the first equation is called the pivot equation and a_{11} is called the first pivot a the end of the first stage, the augumented matrix (3) becomes

$$-\frac{a'_{32}}{a'_{22}}\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b_2 \\ 0 & a'_{32} & a'_{33} & b_3 \end{bmatrix}$$

where $a'_{22}, a'_{23}, \ldots$ are all changed elements. a'_{22} is new pivot and multiplier is $-\frac{a'_{32}}{a'_{22}}$

at the end of the second stage we have upper triangle system

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b_2 \\ 0 & 0 & a''_{33} & b_3 \end{bmatrix}$$

From which the values of x_1, x_2 and x_3 can be obtained by back substitution.

Gaussian Elimination method Sums:

1. Solve the following system of Gauss Elimination method.

$$x_1 - x_2 + x_3 = 1 \longrightarrow ①$$
$$-3x_1 + 2x_2 - 3x_3 = -6 \longrightarrow ②$$
$$2x_1 - 5x_2 + 4x_3 = 5 \longrightarrow ③$$

Sol:

Step-1:

Write a given system in augmentation matrix form

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{bmatrix}$$

Step-2:

$R_2 = R_2 + 3R_1 \Rightarrow -3 + 3(1) = -3 + 3 = 0$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 2 & -5 & 4 & 5 \end{bmatrix} R_2 \to R_2 + 3R_1$$

$$\begin{array}{lrrrr} R_2 \to & -3 & 2 & -3 & -6 \\ 3R_1 \to & 3 & -3 & 3 & 3 \\ \hline & 0 & -1 & 0 & -3 \end{array}$$

Step-3:

$R_3 = R_3 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & -3 & 2 & 3 \end{bmatrix} R_3 \to R_3 - 2R_1$$

$$\begin{array}{lrrrr} R_3 \to & 2 & -5 & 4 & 5 \\ 2R_1 \to & 2 & -2 & 2 & 2 \\ \hline & 0 & -3 & 2 & 3 \end{array}$$

Step-4:

$R_3 = R_3 - 3R_2$

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 2 & 12 \end{bmatrix} R_3 \to R_3 - 3R_2$$

$$\begin{array}{lrrrr} R_3 \to & 0 & -3 & 2 & 3 \\ 3R_2 \to & 0 & -3 & 0 & -9 \\ \hline & 0 & 0 & 2 & 12 \end{array}$$

Step-5:

$$x_1 - x_2 + x_3 = 1 \longrightarrow ④$$
$$-x_2 = -3 \longrightarrow ⑤$$
$$2x_3 = 12 \longrightarrow ⑥$$

$⑥ \Rightarrow 2x_3 = 12$

$$x_3 = \frac{12}{2}$$

$$\boxed{x_3 = 6}$$

$⑥ \Rightarrow -x_2 = -3$

$\boxed{x_2 = 3}$

$① \Rightarrow x_1 - x_2 + x_3 = 1$

$x_1 - 3 + 6 = 1$

$x_1 + 3 = 1$

$x_1 = 1 - 3$

$\boxed{x_1 = -2}$

Verification:

Sub $x_1 = -2$; $x_2 = 3$; $x_3 = 6$ in ①

$x_1 - x_2 + x_3 = 1$

$-2 - 3 + 6 = 1$

$-5 + 6 = 1$

$1 = 1$

LHS = RHS

Hence it is proved

2. Solve the following system of Gauss elimination method.

$3x_1 + x_2 + x_3 = 4 \longrightarrow ①$

$x_1 + 4x_2 - x_3 = -5 \longrightarrow ②$

$x_1 + x_2 - 6x_3 = -12 \longrightarrow ③$

Sol:

Step-1:

write a given system in augmented matrix form

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 4 \\ 1 & 4 & -1 & -5 \\ 1 & 1 & -6 & -12 \end{array}\right]$$

Step-2:

$R_2 = 3R_2 - R_1$

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 4 \\ 0 & 11 & -4 & -19 \\ 1 & 1 & -6 & -12 \end{array}\right] R_2 \to 3R_2 - R_1$$

Step-3:

$R_3 = 3R_3 - R_1$

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 4 \\ 0 & 11 & -4 & -19 \\ 0 & 2 & -19 & -40 \end{array}\right] R_3 \to 3R_3 - R_1$$

Step-4:

$R_3 = 11R_3 - 2R_2$

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 4 \\ 0 & 11 & -4 & -19 \\ 0 & 0 & -201 & -402 \end{array}\right] R_3 \to 11R_3 - 2R_2$$

Step-5:

$3x_1 + x_2 + x_3 = 4 \longrightarrow ④$

$11x_2 - 4x_3 = -19 \longrightarrow ⑤$

$-201x_3 = -402 \longrightarrow ⑥$

⑥ $\Rightarrow$ $-201x_3 = -402$

$$201x_3 = 402$$

$$x_3 = \frac{402}{201}$$

$$\boxed{x_3 = 2}$$

⑤ $\Rightarrow$ $11x_2 - 4x_3 = -19$

$$11x_2 - 4(2) = -19$$

$$11x_2 - 8 = -19$$

$$11x_2 = -19 + 8$$

$$11x_2 = -11$$

$$x_2 = \frac{-11}{11}$$

$$\boxed{x_2 = -1}$$

④ $\Rightarrow$ $3x_1 + x_2 + x_3 = 4$

$$3x_1 - 1 + 2 = 4$$

$$3x_1 + 1 = 4$$

$$3x_1 = 4 - 1$$

$$3x_1 = 3$$

$$x_1 = \frac{3}{3}$$

$$\boxed{x_1 = 1}$$

Verification:

Sub $x_1 = 1$; $x_2 = -1$; $x_3 = 2$ in ①

$$3x_1 + x_2 + x_3 = 4$$

$$3(1) - 1 + 2 = 4$$

$$3 - 1 + 2 = 4$$

Jacobi's (or) Gauss Jacobi's Iteration Method

Let the system of simuntaneous equation be

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\} \longrightarrow (1)$$

This system of equation can also be written as

$$\left.\begin{aligned} x &= \frac{1}{a_1}(d_1 - b_1y - c_1z) \\ y &= \frac{1}{b_2}(d_2 - a_2x - c_2z) \\ z &= \frac{1}{c_3}(d_3 - a_3x - b_3y) \end{aligned}\right\} \longrightarrow (2)$$

Let the first approximation be x_0, y_0 and z_0 in we got

$$x_1 = \frac{1}{a_1}(d_1 - b_1y_0 - c_1z_0)$$

$$y_1 = \frac{1}{b_2}(d_2 - a_2x_0 - c_2z_0)$$

$$z_1 = \frac{1}{c_3}(d_3 - a_3x_0 - b_3y_0)$$

Substituting the values of x_2, y_2 and z_2 we got third approximation x_3, y_3 and z_3. This process may be repeated till the difference between two consecutive approximation is negligible

$$x_2 = \frac{1}{a_1}(d_1 - b_1y_1 - c_1z_1)$$

$$y_2 = \frac{1}{b_2}(d_2 - a_2x_1 - c_2z_1)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3x_1 - b_3y_1)$$

Gauss Seidal Iteration Method:

Let the given system of equation be

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = c_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = c_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n = c_3$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots + a_{nn}x_n = c_n$$

Such a system often amenable to an iteration process which the system is first rewritten in the form

$$x_1 = \frac{1}{a_{11}}(c_1 - a_{12}x_2 - a_{13}x_3 - \cdots - a_{1n}x_n) \longrightarrow (1)$$

$$x_2 = \frac{1}{a_{22}}(c_2 - a_{21}x_1 - a_{23}x_3 - \cdots - a_{2n}x_n) \longrightarrow (2)$$

$$x_3 = \frac{1}{a_{33}}(c_3 - a_{31}x_1 - a_{32}x_2 - \cdots - a_{3n}x_n) \longrightarrow (3)$$

$$x_n = \frac{1}{a_{nn}}(c_n - a_{n1}x_1 - a_{n2}x_2 - \cdots - a_{nn-1}x_{n-1})$$

First let us assume that, $x_2 = x_3 \cdots x_n = 0$ and find x, let it be x_1* putting x_1* for and $x_3 = x_4 \cdots x_n = 0$ in (2) we get the value for x_2 and it let be x_2* putting x_1* for and x_2* for x_2 and $x_3 = x_4 \cdots x_n = 0$ in (3) get value for x_3 and it let be x_3* in may we can find the first approximation values for $x_1, x_2, \ldots x_n$

Similarly we can find the better approximate values of $x_1, x_2, \cdots x_n$ by using the relation

$$x_1^* = \frac{1}{a_{11}}(c_1 - a_{12}x_2 - a_{13}x_3 \ldots a_{1n}x_n)$$

$$x_2^* = \frac{1}{a_{22}}(c_2 - a_{21}x_1^* \; a_{23}x_3 \ldots a_{2n}x_n)$$

$$x_3^* = \frac{1}{a_{33}}(c_3 - a_{31}x_1^* \; a_{32}x_2 \ldots a_{3n}x_n)$$

$$x_n^* = \frac{1}{a_{nn}}(c_n - a_{n1}x_1 - a_{n2}x_2 + \ldots - a_{n3}x_3 \ldots a_{nn-1}x_{n-1})$$

Jacobi Iteration Method Sums:

1. Solve the following equation using Jacobi iteration method.

$$20x + y - 2z = 17 \longrightarrow ①$$
$$3x + 20y - z = -18 \longrightarrow ②$$
$$2x - 3y + 20z = 25 \longrightarrow ③$$

Sol:

The given equation can be written as

$① \Rightarrow 20x + y - 2z = 17$

$$20x = 17 - y + 2z$$

$$x = \frac{1}{20}[17 - y + 2z]$$

$② \Rightarrow 3x + 20y - z = -18$

$$20y = z - 18 - 3x$$

$$y = \frac{1}{20}[z - 18 - 3x]$$

$③ \Rightarrow 2x - 3y + 20z = 25$

$$20z = 3y - 2x + 25$$

$$z = \frac{1}{20}[3y - 2x + 25]$$

First approximation:

$$x_1 = \frac{1}{20}[17 - y_0 + 2z_0]$$

$$y_1 = \frac{1}{20}[-18 - 3x_0 + z_0]$$

$$z_1 = \frac{1}{20}[3y_0 - 2x_0 + 25]$$

put $x_0 = y_0 = z_0 = 0$

$$x_1 = \frac{1}{20}[17 - 0 + 0] = \frac{17}{20} \Rightarrow \boxed{x_1 = 0.85}$$

$$y_1 = \frac{1}{20}[-18 - 0 + 0] = \frac{-18}{20} \Rightarrow \boxed{y_1 = -0.9}$$

$$z_1 = \frac{1}{20}[0 - 0 + 25] = \frac{25}{20} \Rightarrow \boxed{z_1 = 1.25}$$

Second approximation:

$$x_2 = \frac{1}{20}[17 - y_1 + 2z_1]$$

$$y_2 = \frac{1}{20}[-18 - 3x_1 + z_1]$$

$$z_2 = \frac{1}{20}[3y_1 - 2x_1 + 25]$$

put, $x_1 = 0.85$; $y_1 = -0.9$; $z_1 = 1.25$

$$x_2 = \frac{1}{20}[17 + 0.9 + 2(1.25)] = \frac{1}{20}[17 + 0.9$$

$$\boxed{x_2 = 1.02}$$

$$y_2 = \frac{1}{20}\left[-18 - 3(0.85) + 1.25\right] = \frac{1}{20}\left[-18 - 2.55 + \dots\right.$$

$$\boxed{y_2 = -0.965}$$

$$z_2 = \frac{1}{20}\left[3(-0.9) - 2(0.85) + 25\right] = \frac{1}{20}\left[-2.7 - 1.7 + 25\dots\right.$$

$$\boxed{z_2 = 1.03}$$

Third approximation:

$$x_3 = \frac{1}{20}\left[17 - y_2 + 2z_2\right]$$

$$y_3 = \frac{1}{20}\left[-18 - 3x_3 + z_2\right]$$

$$z_3 = \frac{1}{20}\left[25 - 2x_3 + 3y_3\right]$$

Put, $x_2 = 1.02$; $y_2 = -0.965$; $z_2 = 1.03$

$$x_3 = \frac{1}{20}\left[17 + 0.965 + 2(1.03)\right]$$

$$\boxed{x_3 = 1.00125}$$

$$y_3 = \frac{1}{20}\left[-18 - 3(1.02) + 1.03\right]$$

$$\boxed{y_3 = -1.0016}$$

$$z_3 = \frac{1}{20}\left[25 - 2(1.02) + 3(-0.965)\right]$$

$$\boxed{z_3 = 1.00325}$$

Fourth approximation

$$x_4 = \frac{1}{20}\left[17 - y_3 + 2z_3\right]$$

$$y_4 = \frac{1}{20}\left[-18 - 3x_3 + z_3\right]$$

$$z_4 = \frac{1}{20}\left[25 - 2x_3 + 3y_3\right]$$

Put, $x_3 = 1.00125$; $y_3 = -1.0016$; $z_3 = 1.00325$

$$x_4 = 1/20\left[17 + 1.0016 + 2(1.00325)\right]$$

$$\boxed{x_4 = 1.0004}$$

$$y_4 = \frac{1}{20}\left[-18 - 3(1.00125) + 1.00325\right]$$

$$\boxed{y_4 = -1.000025}$$

$$z_4 = \frac{1}{20}\left[25 - 2(1.00125) + 3(-1.0016)\right]$$

$$\boxed{z_4 = 0.99965}$$

Fifth approximation:

$$x_5 = \frac{1}{20}\left[17 - y_4 + 2z_4\right]$$

$$y_5 = \frac{1}{20}\left[-18 - 3x_4 + z_4\right]$$

$$z_5 = \frac{1}{20}\left[25 - 2x_4 + 3y_4\right]$$

Put, $x_4 = 1.0004$; $y_4 = -1.000025$; $z_4 =$

$$x_5 = \frac{1}{20}\left[17 + 1.000025 + 2(0.99965)\right]$$

$$\boxed{x_5 = 0.9999}$$

$y_5 = \frac{1}{20}[-18 - 3(1.0004) + 0.99965]$

$\boxed{y_5 = -1.0000775}$

$z_5 = \frac{1}{20}[25 - 2(1.0004) + 3(-1.000025)]$

$\boxed{z_5 = 0.9999}$

From the 4 and 5 approximation $x = 1, y = 1, z = 1$

---

2. Solve the System of equation Gauss Jaccobi iteration method.

$28x + 4y - z = 32 \longrightarrow (1)$

$x + 3y + 10z = 24 \longrightarrow (2)$

$2x + 17y + 4z = 35 \longrightarrow (3)$

Sol:

Interchange (2) & (3) we get diaganally dominated.

$28x + 4y - z = 32 \longrightarrow (1)$

$2x + 17y + 4z = 35 \longrightarrow (2)$

$x + 3y + 10z = 24 \longrightarrow (3)$

$(1) \Rightarrow 28x + 4y - z = 32$

$28x = 32 - 4y + z$

$x = \frac{1}{28}[32 - 4y + z]$

$(2) \Rightarrow 2x + 17y + 4z = 35$

$17y = 35 - 2x - 4z$

$y = \frac{1}{17}[35 - 2x - 4z]$

$(3) \Rightarrow x + 3y + 10z = 24$

$10z = 24 - x - 3y$

$z = \frac{1}{10}[24 - x - 3y]$

First approximation:

$x_1 = \frac{1}{28}[32 - 4y_0 + z_0]$

$y_1 = \frac{1}{17}[35 - 2x_0 - 4z_0]$

$z_1 = \frac{1}{10}[24 - x_0 - 3y_0]$

put, $x_0 = y_0 = z_0 = 0$

$x_1 = \frac{1}{28}[32] \Rightarrow \boxed{x_1 = 1.1438}$

$y_1 = \frac{35}{17} \Rightarrow \boxed{y_1 = 2.059}$

$z_1 = \frac{24}{10} \Rightarrow \boxed{z_1 = 2.4}$

second approximation:

	x	y	z
	$\frac{1}{28}[32 - 4y + z]$	$\frac{1}{17}[35 - 2x - 4z]$	$\frac{1}{10}[24 - x -$
x_1	1.143	2.059	2.4
x_2	0.934	1.359	1.668
x_3	1.008	1.556	1.898
x_4	0.988	1.4936	1.832
x_5	0.9949	1.5115	1.85
x_6	0.993	1.5057	1.84

From the (5) & (6) approximation $x = 1, y =$

Gauss Seidal Method :

2 mark

Example - 1

The co-efficient matrix $\begin{pmatrix} 8 & -3 & 2 \\ 6 & 3 & 12 \\ 4 & 11 & -1 \end{pmatrix}$ can be made diagnally dominated by interchanging second and third rows. $\begin{pmatrix} 8 & -3 & 2 \\ 4 & 11 & -1 \\ 6 & 3 & 12 \end{pmatrix}$

Example - 2

consider the system of equation

$x_1 + 7x_2 - x_3 = 3$

$5x_1 + x_2 + x_3 = 9$

$-3x_1 + 2x_2 + 7x_3 = 17$

The co-efficient $\begin{pmatrix} 1 & 7 & -1 \\ 5 & 1 & 1 \\ -3 & 2 & 7 \end{pmatrix}$ is not diagnally dominated.

But the given system can be rearranged.

$7x_2 + x_1 - x_3 = 3$

$x_2 + 5x_1 + x_3 = 9$

$2x_2 - 3x_1 + 7x_3 = 17$

$\begin{pmatrix} 7 & 1 & -1 \\ 1 & 5 & 1 \\ 2 & -3 & 7 \end{pmatrix}$

Now, the co-efficient matrix is a diagnally dominated.

Gauss Seidal Method Sum:

1) Solve the system of equation

$$8x-y+z-18=0$$
$$2x+5y-2z-3=0$$
$$x+y-3z+6=0$$

using Gauss seidal iteration method.

Sol:

The given equation is

$$8x-y+z=18 \longrightarrow (1)$$
$$2x+5y-2z=3 \longrightarrow (2)$$
$$x+y-3z=-6 \longrightarrow (3)$$

The co-efficient matrix $\begin{pmatrix} 8 & -1 & 1 \\ 2 & 5 & -2 \\ 1 & 1 & -3 \end{pmatrix}$ is diagonally dominated

$(1) \Rightarrow 8x-y+z=18$

$$8x = 18+y-z$$
$$x = \frac{1}{8}[18+y-z] \longrightarrow (4)$$

$(2) \Rightarrow 2x+5y-2z=3$

$$5y = 3-2x+2z$$
$$y = \frac{1}{5}[3-2x+2z] \longrightarrow (5)$$

$(3) \Rightarrow x+y-3z=-6$

$$-3z = [-6-x-y]$$
$$z = -\frac{1}{3}[-6-x-y]$$
$$z = \frac{1}{3}[6+x+y] \longrightarrow (6)$$

1st iteration:

putting $y=0$, $z=0$ in (4) we get

$$x = \frac{18}{8}$$
$$\boxed{x = 2.25}$$

putting $x=2.25$, $z=0$ in (5) we get

$$y = \frac{1}{5}[3-2(2.25)+0]$$
$$\boxed{y = -0.3}$$

putting $x=2.25$, $y=-0.3$ in (6) we get

$$z = \frac{1}{3}[6+2.25-0.3]$$
$$\boxed{z = 2.65}$$

2nd iteration:

putting $y=-0.3$, $z=2.65$ in (4) we get

$$x = \frac{1}{8}[18-0.3-2.65]$$
$$\boxed{x = 1.8813}$$

putting $x=1.8813$, $z=2.65$ in (5) we

$$y = \frac{1}{5}[3-2(1.8813)+2(2.65)]$$
$$\boxed{y = 0.9075}$$

putting $x=1.8813$, $y=0.9075$ in (6)

$$z = \frac{1}{3}[6+1.8813+0.9075]$$
$$\boxed{z = 2.9296}$$

IIIrd Iteration:

putting $y = 0.9935$, $z = 2.9996$ in ② we get

$$x = \frac{1}{8}\left[18 + 0.9935 - 2.9996\right]$$

$$\boxed{x = 1.9975}$$

putting $x = 1.9978$, $z = 2.9996$ in ③ we get

$$y = \frac{1}{5}\left[3 - 2(1.9978) + 2(2.9996)\right]$$

$$\boxed{y = 0.97294}$$

putting $x = 1.997$, $y = 0.97294$ in ④ we get

$$z = \frac{1}{3}\left[6 + 1.997 + 0.97294\right]$$

$$\boxed{z = 2.9900}$$

Therefore IIIrd Iteration we get $x = 1.9978$, $y = 0.97294$, $z = 2.990$

IVth Iteration:

putting $y = 0.9729$, $z = 2.9900$ in ② we get

$$x = \frac{1}{8}\left[18 + 0.9729 - 2.9900\right]$$

$$\boxed{x = 1.9978}$$

putting $x = 1.9978$, $z = 2.9900$ in ③ we get

$$y = \frac{1}{5}\left[3 - 2(1.9978) + 2(2.9900)\right]$$

$$\boxed{y = 0.9968}$$

putting $x = 1.9978$, $y = 0.9968$ in ④ we get

$$z = \frac{1}{3}\left[6 + 1.9978 + 0.9968\right]$$

$$\boxed{z = 2.9982}$$

Therefore IVth Iteration we get $x = 1.9978$, $y = 0.9968$, $z = 2.9982$

Vth Iteration:

putting $y = 0.9968$, $z = 2.9982$ in ② we get

$$x = \frac{1}{8}\left[18 + 0.9968 - 2.9982\right]$$

$$\boxed{x = 1.9998}$$

putting $x = 1.9998$, $z = 2.99822$ in ③ we get

$$y = \frac{1}{5}\left[3 - 2(1.9998) + 2(2.99822)\right]$$

$$\boxed{y = 0.99936}$$

putting $x = 1.9998$, $y = 0.99936$ in ④ we get

$$z = \frac{1}{3}\left[6 + 1.9998 + 0.99936\right]$$

$$\boxed{z = 2.9997}$$

Therefore Vth Iteration we get $x = 1.9998$, $y = 0.99936$, $z = 2.9997$

VIth Iteration:

putting $y = 0.99936$, $z = 2.9997$ in ② we get

$$x = \frac{1}{8}\left[18 + 0.99936 - 2.9997\right]$$

$$\boxed{x = 2.00}$$

putting $x = 2.00$, $z = 2.9997$ in ③ we get

$$y = \frac{1}{5}\left[3 - 2(2.00) + 2(2.9997)\right]$$

$$\boxed{y = 0.9999}$$

putting $z = 2.00$, $y = 0.9998$ in ①

$$x = \frac{1}{3}[6 + 2.00 + 0.9998]$$

$$\boxed{x = 2.9999}$$

Therefore VI^{th} iteration we get $x = 2.00$,
$y = 0.9998$, $z = 2.9999$

From the V and VI iteration we get

$x = 2.00$, $y = 0.999$, $z = 2.999$

---

2. Solve the system of equation using Gauss seidal iteration method.

$$2x + y + z = 4 \longrightarrow ①$$

$$x + 2y + z = 4 \longrightarrow ②$$

$$x + y + 2z = 4 \longrightarrow ③$$

Sol:

① $\Rightarrow$ $x = \frac{1}{2}[4 - y - z]$

② $\Rightarrow$ $y = \frac{1}{2}[4 - x - z]$

③ $\Rightarrow$ $z = \frac{1}{2}[4 - x - y]$

	$x = \frac{1}{2}[4-y-z]$	$y = \frac{1}{2}[4-x-z]$	$z = \frac{1}{2}[4-x-y]$
x_1	2	1	0.5
x_2	1.25	1.125	0.8125
x_3	1.03	1.07875	0.9456
x_4	0.9878	1.0333	0.98945
x_5	0.9886	1.0109	1.00025
x_6	0.9944	1.0026	1.0015

x_7	0.99795	1.00027	1.0008
x_8	0.9994	0.9999	1.00035
x_9	0.9998	0.9999	1.00015

From ⑧ & ⑨ iteration, we get

$x=1, y=1, z=1$

Gauss seidal method Home work Sums:

1. $4x+2y+z=14$ Ans: $x=2.00$
$x+5y-z=10$ $y=1.99$
$x+y+8z=20$ $z=1.999$

2. $28x+4y-z=32$ Ans: $x=0.994$
$x+3y+10z=24$ $y=1.507$
$2x+17y+4z=35$ $z=1.849$

3. $8x-3y+2z=20$ Ans: $x=3.02$
$6x+3y+12z=35$ $y=1.9845$
$4x+11y-z=33$ $z=0.91$

4. $30x-2y+3z=75$ Ans: $x=2.5796$
$x+17y-2z=48$ $y=2.79676$
$2x+2y+18z=30$ $z=1.0692$

5. $9x_1+2x_2+4x_3=20$ Ans: $x_1=2.737$
$x_1+10x_2+4x_3=6$ $x_2=0.987$
$2x_1+4x_2+10x_3=-15$ $x_3=-1.652$

6. $10x_1-2x_2-x_3-x_4=3$ Ans: $x_1=1$
$-2x_1+10x_2-x_3-x_4=15$ $x_2=2$
$-x_1-x_2+10x_3-2x_4=27$ $x_3=3$
$-x_1-x_2-2x_3+10x_4=-9$ $x_4=0$