UNIT - III

NUMERICAL INTEGRATION :

The general problem of numerical integration ~~ke~~ may be stated as follows given a set of data points $(x_0, y_0), (x_1, y_1), \ldots (x_n, y_n)$ of a function $y = f(x)$

Where $f(x)$ is not known explity. It is required to compute the value of the defined integral

$$I = \int_a^b y \, dx$$

Let, the interval $[a, b]$ be divided into n equal Subinterval such that $a = x_0 < x_1 < x_2 < \cdots < x_n = b$ clear by $x_n = x_0 + nh$. Hence the integral becomes

$$I = \int_{x_0}^{x_n} y \, dx$$

Approximation y by Newton's forward difference formula, we obtain

$$I_n = \int_{x_0}^{x_n} \left[y_0 + P\Delta y_0 + \frac{P(P-1)}{2} \Delta^2 y_0 + \frac{P(P-1)(P-2)}{6} \Delta^3 y_0 + \cdots \right] d$$

Since $x = x_0 + Ph$, $dx = h\,dp$ and hence the above integral become,

$$I = h \int_0^h \left[y_0 + P\Delta y_0 + \frac{P(P-1)}{2} \Delta^2 y_0 + \frac{P(P-1)(P-2)}{6} \Delta^3 y_0 \cdots \right] dp$$

Which gives on simplification.

Formula:

$$\int_{x_0}^{x_n} y\,dx = nh\left[y_0 + \frac{n}{2}\Delta y_0 + \frac{n(2n-3)}{12}\Delta^2 y_0 + \frac{n(n-2)^2}{24}\Delta^3 y_0 + \cdots\right]$$

From the general formula we can obtain different integration formula by putting n = 1, 2, 3, 4 etc. We derive here few of those formula but it should be remarked that the Trapezoidal and Simpson's 1/3 rules are found to give sufficient accuracy for use in practical problem.

i) Trapezoidal problem

Setting n = 1 in the general formula all difference higher than the first will become zero and we obtain x,

$$\int_{x_0}^{x_1} y\,dx = h\left[y_0 + \frac{1}{2}\Delta y_0\right] = h\left[y_0 + \frac{1}{2}(y_1 - y_0)\right] = \frac{h}{2}\left[y_0 + y_1\right]$$

for the next interval $[x_1, x_2]$ we deduce similarly,

$$\int_{x_1}^{x_2} y\,dx = \frac{h}{2}\left[y_1 + y_2\right]$$

and, so on list interval $[x^{n-1}, x^n]$ we have

$$\int_{x^{n-1}}^{x^n} y\,dx = \frac{h}{2}\left[y_{n-1} + y_n\right]$$ combining all those expressions, we obtain the rule

Formula:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + \cdots y_{n-1}) + y_n\right]$$

Which is known as the Trapezoidal rule

ii) Simpson's 1/3 rule: (or) Simpson's Rule

The rule is obtained by putting n = 2 in equation by replacing the curve by n/2 arcs of 2nd degree polynomial on parabola we have them

$$\int_{x_0}^{x_2} y\,dx = 2h\left[y_0 + \Delta y_0 + \frac{1}{6}\Delta^2 y_0\right]$$

$$= \frac{h}{3}\left[y_0 + 4y_1 + y_2\right]$$

Similarly,

$$\int_{x_2}^{x_4} y\,dx = \frac{h}{3}\left[y_2 + 4y_3 + y_4\right]$$

and finally

$$\int_{x_{n-2}}^{x_n} y\,dx = \frac{h}{3}\left[y_{n-2} + 4y_{n-1} + y_n\right]$$

Summing up we obtain Simpson's 1/3 formula

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + \cdots + y_{n-1}) + 2(y_2 + y_4 + y_6 + \cdots + y_{n-2}) + y_n\right]$$

Which is known as Simpson's Rule (or) simply Simpson's rule

iii) Simpson's 3/8 rule:

Setting x = 3 in (5.29) we observe that all differences higher than the 3rd will become zero and we obtain

$$\int_{x_0}^{x_3} y\,dx = 3h\left[y_0 + \frac{3}{2}\Delta y_0 + \frac{3}{4}\Delta^2 y_0 + \frac{1}{8}\Delta^3 y_0\right]$$

$$= 3h\left[y_0 + \tfrac{3}{2}(y_1 - y_0) + \tfrac{3}{4}(y_2 - 2y_1 + y_0) + \tfrac{3}{8}(y_3 - 3y_2 + 3y_1 - y_0)\right]$$

$$= \frac{3h}{8}\left[y_0 + 3y_1 + 3y_2 + y_3\right]$$

III^{ly},

$$\int_{x_3}^{x_6} y\,dx = \frac{3h}{8}\left[y_3 + 3y_4 + 3y_5 + 3y_6\right]$$ and so on. Summing up all these we obtain

$$\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[(y_0 + 3y_1 + 3y_2 + y_3) + (y_3 + 3y_4 + 3y_5 + 3y_6) + \cdots + (y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n)\right]$$

$$= \frac{3h}{8}\left[y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + 2y_6 + \cdots + 2y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n\right]$$

(or)

$$\boxed{\int_{x_0}^{x_n} y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5 + \cdots) + 2(y_3 + y_6 + y_9 + \cdots) + y_n\right]}$$

This is called as Simpson's 3/8 rule

1. Evaluate $\int_{0.6}^{2} y\,dx$ using (i) Trapezoidal Rule (ii) Simpson's 1/3 Rule (iii) Simpson's 3/8 Rule. The following table

x	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
y	1.23	1.58	2.03	4.32	6.25	8.36	10.23	12.45
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7

Sol:

Here $h = 0.2$

i) Trapezoidal Rule:

$$\int_{0.6}^{2} y\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6) + y_7\right]$$

$$= \frac{0.2}{2}\left[1.23 + 2(1.58 + 2.03 + 4.32 + 6.25 + 8.36 + 10.23) + 12.45\right]$$

$$= 0.1\left[1.23 + 2(32.77) + 12.45\right]$$

$$= 0.1\left[79.22\right]$$

$$\int_{0.6}^{2} y\,dx = 7.922$$

ii) Simpson's 1/3 rule:

$$\int_{0.6}^{2} y\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4 + y_6) + y_7\right]$$

$$= \frac{0.2}{3}\left[1.23 + 4(1.58 + 4.32 + 8.36) + 2(2.03 + 6.25 + 10.23) + 12.45\right]$$

$= 0.067 \left[1.23 + 4(19.96) + 2(18.00) + 12.49\right]$

$= 0.067(107.74)$

$= 7.21858$

iii) Simpson's 3/8 Rule:

$$\int_{0.6}^{2} y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3 + y_6) + y_n\right]$$

$$= \frac{3(0.2)}{8}\left[1.23 + 3(1.58 + 2.03 + 6.49 + 8.36) + 2(4.32 + 10.23) + 12.49\right]$$

$$= 0.075\left[1.23 + 3(18.46) + 2(14.55) + 12.49\right]$$

$$= 0.075(97.44)$$

$$\int_{0.6}^{2} y\,dx = 7.308$$

2. Evaluate $\int_{1}^{7} y\,dx$ using i) Trapezoidal Rule ii) Simpson's 1/3 Rule iii) Simpson's 3/8 Rule

The following table

x	1	2	3	4	5	6	7
y	2.105	2.808	3.814	4.604	5.857	7.341	9.467

Sol:

Here $h = 1$

i) Trapezoidal Rule:

$$\int_{1}^{7} y\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5) + y_n\right]$$

$$= \frac{1}{2}\left[2.105 + 2(2.808 + 3.814 + 4.604 + 5.857 + 7.341) + 9.467\right]$$

$$= \frac{1}{2}\left[2.105 + 48.848 + 9.467\right]$$

$$= \frac{1}{2}\left[60.42\right]$$

$$\int_{1}^{7} y\,dx = 30.21$$

ii) Simpson's 1/3 Rule:

$$\int_{1}^{7} y\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) + y_n\right]$$

$$= \frac{1}{3}\left[2.105 + 4(2.808 + 4.604 + 7.341) + 2(3.814 + 5.857) + 9.467\right]$$

$$= \frac{1}{3}\left[2.105 + 59.012 + 18.942 + 9.467\right]$$

$$= \frac{1}{3}\left[90.326\right]$$

$$\int_{1}^{7} y\,dx = 30.10$$

iii) Simpson's 3/8 Rule:

$$\int_1^7 y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1+y_2+y_4+y_5) + 2(y_3) + y_n\right]$$

$$= \frac{3}{8}\left[2.105 + 3(2.808 + 3.614 + 5.887 + 7.541) + 2(4.604) + 9.467\right]$$

$$= \frac{3}{8}\left[2.105 + 59.46 + 9.208 + 9.467\right]$$

$$= \frac{3}{8}\left[80.24\right]$$

$$\int_1^7 y\,dx = 30.09$$

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3. Evaluate $\int_4^{5.2} \log_e x$ using i) Trapezoidal Rule ii) Simpson's 1/3 Rule iii) Simpson's 3/8 Rule

The following table

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
y	1.3862	1.4350	1.4816	1.5260	1.5682	1.6094	1.6486
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Sol:

Here, $h = 0.2$

i) Trapezoidal Rule:

$$\int_4^{5.2} \log_e x = \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3+y_4+y_5) + y_n\right]$$

$$= \frac{0.2}{2}\left[1.3862 + 2(1.4350 + 1.4816 + 1.5260 + 1.5682 + 1.6094) + 1.6486\right]$$

$$= \frac{0.2}{2}\left[1.3862 + 2(7.6202) + 1.6486\right]$$

$$= 0.1(18.2752)$$

$$= 1.8275$$

ii) Simpson's 1/3 Rule:

$$\int_4^{5.2} \log_e x = \frac{h}{3}\left[y_0 + 4(y_1+y_3+y_5) + 2(y_2+y_4) + y_n\right]$$

$$= \frac{0.2}{3}\left[1.3862 + 4(1.4350 + 1.5260 + 1.6094) + 2(1.4816 + 1.5682) + 1.6486\right]$$

$$= \frac{0.2}{3}\left[1.3862 + 4(4.5704) + 2(3.0498) + 1.6486\right]$$

$$= 0.067\left[1.3862 + 18.2816 + 6.0996 + 1.6486\right]$$

$$= 0.067\left[27.4160\right]$$

$$= 1.83687$$

iii) Simpson's 3/8 Rule:

$$\int_4^{5.2} \log_e x = \frac{3h}{8}\left[y_0 + 3(y_1+y_2+y_4+y_5) + 2(y_3) + y_n\right]$$

$$= \frac{3(0.2)}{8}\left[1.3862 + 3(1.4350 + 1.4816 + 1.5682 + 1.6094) + 2(1.5260) + 1.6486\right]$$

10. A river is 80 feet wide the department (d) increase at a distance x feet from the bank is given by the following by using Simpson's 1/3 rule

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Sol:

Here, $h = 10$

Simpson's 1/3 Rule

$$\int_0^{80} y\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6) + y_8\right]$$

$$= \frac{10}{3}\left[0 + 4(4 + 9 + 15 + 8) + 2(7 + 12 + 14) + 3\right]$$

$$= \frac{10}{3}\left[144 + 66 + 3\right]$$

$$= 3.3333\left[213\right]$$

$$= 709.999$$

$$\int_0^{80} y\,dx = 710 \text{ Square feet}$$

Home work Sums

1. Using the data given below find the value of $\int_1^9 y\,dx$ using i) Trapezoidal Rule ii) Simpson's 1/3 rule iii) Simpson's 3/8 rule The following table

x	1	2	3	4	5	6	7	8	9
y	2.061	2.312	2.891	3.106	3.616	4.721	6.103	7.690	9.942

Ans: i) 36.7545

ii) 36.7619

iii) 35.6448

2. Using the data given below find the value of $\int_0^1 x^2\,dx$ using i) Trapezoidal Rule ii) Simpson's 1/3 rule iii) Simpson's 3/8 rule The following table

x	0	0.25	0.50	0.75	1
$y = x^2$	0	0.0625	0.25	0.5625	1

Ans: i) 0.34375

ii) 0.3333

iii) 0.3

HOME WORK SUMS ANSWERS

4. Using the data given below find the value of $\int_1^9 y\,dx$ using i) Trapezoidal rule ii) Simpson's $1/3$ rule iii) Simpson's $3/8$ rule the following table

x	1	2	3	4	5	6	7	8	9
y	2.061	2.312	2.891	3.106	3.670	4.721	6.103	7.980	9.942

$y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6 \quad y_7 \quad y_n$

Sol:

Here $h = 1$

i) Trapezoidal Rule

$$\int_1^9 y\,dx = \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3+y_4+y_5+y_6+y_7) + y_n\right]$$

$$= \frac{1}{2}\left[2.061 + 2(2.312 + 2.891 + 3.106 + 3.670 + 4.721 + 6.103 + 7.980) + 9.942\right]$$

$$= \frac{1}{2}\left[2.061 + 2(30.783) + 9.942\right]$$

$$= \frac{1}{2}\left[2.061 + 61.566 + 9.942\right]$$

$$= 0.5\,(73.569)$$

$$\int_1^9 y\,dx = 36.7845$$

ii) Simpson's $1/3$ rule:

$$\int_1^9 y\,dx = \frac{1}{3}\left[y_0 + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6) + y_n\right]$$

$$= \frac{1}{3}\left[2.061 + 4(2.312 + 3.106 + 4.721 + 7.980) + 2(2.891 + 3.670 + 6.103) + 9.942\right]$$

$$= 0.3333\left[2.061 + 4(18.089) + 2(12.664) + 9.942\right]$$

$$= 0.3333\left[2.061 + 72.356 + 25.328 + 9.942\right]$$

$$= 0.3333\,[109.687]$$

$$\int_1^9 y\,dx = 36.525771$$

iii) Simpson's $3/8$ rule

$$\int_1^9 y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1+y_2+y_4+y_5+y_7) + 2(y_3+y_6) + y_n\right]$$

$= \frac{3}{8}$ [illegible]

[illegible]

$= 0.375$ [illegible]

$= 0.375$ [illegible]

$= 0.375\,($[illegible]$)$

$\int_1^9 y\,dx =$ [illegible]

5. Using the data given below find the value of $\int_0^1 x^2\,dx$ using i) Trapezoidal rule ii) Simpson's $1/3$ rule iii) Simpson's $3/8$ rule the following table

x	0	0.25	0.50	0.75	1
$y = x^2$	0	0.0625	0.25	0.5625	1

$y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_n$

Sol:

Here $h = 0.25$

i) Trapezoidal Rule:

$$\int_0^1 x^2\,dx = \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3) + y_n\right]$$

$$= \frac{0.25}{2}\left[0 + 2(0.0625 + 0.25 + 0.5625) + 1\right]$$

$$= 0.125\left[0 + 2(0.875) + 1\right]$$

$$= 0.125\left[0 + 1.75 + 1\right]$$

$$= 0.125\,[2.75]$$

$$\int_0^1 x^2\,dx = 0.34375$$

ii) Simpson's 1/3 rule:

$$\int_0^1 x^2\,dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3) + 2(y_2) + y_n\right]$$

$$= \frac{0.25}{3}\left[0 + 4(0.0625 + 0.5625) + 2(0.25) + 1\right]$$

$$= 0.0833\left[0 + 4(0.625) + 2(0.25) + 1\right]$$

$$= 0.0833\left[0 + 2.5 + 0.5 + 1\right]$$

$$= 0.0833\left[4\right]$$

$$\int_0^1 x^2\,dx = 0.3332$$

iii) Simpson's 3/8 rule:

$$\int_0^1 x^2\,dx = \frac{3(0.25)}{8}\left[y_0 + 3(y_1+y_2) + 2(y_3) + y_n\right]$$

$$= 0.09375\left[0 + 3(0.0625 + 0.25) + 2(0.5625) + 1\right]$$

$$= 0.09375\left[0 + 3(0.3125) + 2(0.5625) + 1\right]$$

$$= 0.09375\left[0 + 0.9375 + 1.125 + 1\right]$$

$$= 0.09375\left[3.0625\right]$$

$$= 0.28710$$

$$\int_0^1 x^2\,dx = 0.3$$

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7. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with $h = 0.2$ hence the decimal value of π

Sol:

Here $h = 0.2$

The value of the function $y = \frac{1}{1+x^2}$ for each point of the subdivision are given below

x	0	0.2	0.4	0.6	0.8	1
$y = \frac{1}{1+x^2}$	1	0.9615	0.8621	0.7353	0.6098	0.5

Trapezoidal Rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3+y_4) + y_n\right]$$

$$= \frac{0.2}{2}\left[1 + 2(0.9615 + 0.8621 + 0.7353 + 0.6098) + 0.5\right]$$

$$= \frac{0.2}{2}\left[1 + 6.3374 + 0.5\right]$$

$$= \frac{0.2}{2}\left[7.8374\right]$$

$$\int_0^1 \frac{dx}{1+x^2} = 0.78374$$

We know that,

$$\int_0^1 \frac{1}{1+x^2}\,dx = (\tan^{-1})x$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= 45 - 0$$

Home work Sum Answers:

1. Given $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$, $e^4 = 54.60$ using i) Trapezoidal rule ii) Simpson's $\frac{1}{3}$ rule iii) Simpson's $\frac{3}{8}$ rule to find an approximate value of $\int_0^4 e^x dx$ your result with exact of the integral

x	0	1	2	3	4
y	1	2.72	7.39	20.09	54.60
	y_0	y_1	y_2	y_3	y_4

Sol:

Here $h = 1$

i) Trapezoidal rule:

$$\int_0^4 e^x dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_4\right]$$

$$= \frac{1}{2}\left[1 + 2(2.72 + 7.39 + 20.09) + 54.60\right]$$

$$= 0.5\left[1 + 2(30.2) + 54.60\right]$$

$$= 0.5\left[1 + 60.4 + 54.60\right]$$

$$= 0.5\left[116\right]$$

$$= 58$$

$$\int_0^4 e^x dx = 58$$

ii) Simpson's $\frac{1}{3}$ rule:

$$\int_0^4 e^x dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3) + 2(y_2) + y_4\right]$$

$$= \frac{1}{3}\left[1 + 4(2.72 + 20.09) + 2(7.39) + 54.60\right]$$

$$= \frac{1}{3}\left[1 + 4(22.81) + 2(7.39) + 54.60\right]$$

$$= 0.33\left[1 + 91.24 + 14.78 + 54.60\right]$$

$$= 0.33\left[161.62\right]$$

$$= 53.867$$

iii) Simpson's $\frac{3}{8}$ rule:

$$\int_0^4 e^x dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2) + 2(y_3) + y_4\right]$$

$$= \frac{3}{8}\left[1 + 3(2.72 + 7.39) + 2(20.09) + 54.60\right]$$

$$= 0.375\left[1 + 3(10.11) + 2(20.09) + 54.60\right]$$

$$= 0.375\left[1 + 30.33 + 40.18 + 54.60\right]$$

$$= 0.375\left[126.11\right]$$

$$= 47.29125$$

2) $\int_1^2 \sin x \, dx$ by using three rules

x	1	1.2	1.4	1.6	1.8	2.0
y	0.84147	0.9320	0.9854	0.9996	0.9738	0.9092
	y_0	y_1	y_2	y_3	y_4	y_n

Sol:

Here $h = 0.2$

i) Trapezoidal rule:

$$\int_1^2 \sin x \, dx = \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3+y_4) + y_n\right]$$

$$= \frac{0.2}{2}\left[0.84147 + 2(0.9320 + 0.9854 + 0.9996 + 0.9738) + 0.9092\right]$$

$$= 0.1\left[0.84147 + 7.7816 + 0.9092\right]$$

$$= 0.953207$$

$$= 0.95321$$

ii) Simpson's $\frac{1}{3}$ rule:

$$\int_1^2 \sin x \, dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3) + 2(y_2 + y_4) + y_n\right]$$

$$= \frac{0.2}{3}\left[0.84147 + 4(0.9320 + 0.9996) + 2(0.9854 + 0.9738) + 0.9092\right]$$

$$= 0.0666\left[0.84147 + 7.726 + 3.9184 + 0.9092\right]$$

$$= 0.0666\left[13.39601\right]$$

$$= 0.89211$$

iii) Simpson's $3/8$ rule

$$\int_1^2 \sin x \, dx = \frac{3h}{8}\left[y_0 + 3(y_1+y_2+y_4) + 2(y_3) + y_n\right]$$

$$= \frac{3(0.2)}{8}\left[0.84147 + 3(0.9320 + 0.9854 + 0.9738) + 2(0.9995) + 0.9092\right]$$

$$= 0.075\left[0.84147 + 8.6736 + 1.999 + 0.9092\right]$$

$$= 0.075\left[12.42327\right]$$

$$= 0.9317452$$

$$= 0.931$$

Home work sums and answers:

1. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using the three rules $h = 0.25$

Sol:

Here $h = 0.25$

x	0	0.25	0.5	0.75	1
$y = \frac{1}{1+x^2}$	1	0.9411	0.8	0.64	0.5
	y_0	y_1	y_2	y_3	y_n

i) Trapezoidal rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3) + y_n\right]$$

$$= \frac{0.25}{2}\left[1 + 2(0.9411 + 0.8 + 0.64) + 0.5\right]$$

$$= 0.125\left[1 + 2(2.3811) + 0.5\right]$$

$$= 0.125\left[1 + 4.7622 + 0.5\right]$$

$$= 0.125\left[6.2622\right]$$

$$\int_0^1 \frac{dx}{1+x^2} = 0.782775$$

ii) Simpson's $\frac{1}{3}$ rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[y_0 + 4(y_1+y_3) + 2(y_2) + y_n\right]$$

$$= \frac{0.25}{3}\left[1 + 4(0.9411 + 0.64) + 2(0.8) + 0.5\right]$$

$$= 0.0833\left[1 + 4(1.5811) + 2(0.8) + 0.5\right]$$

$$= 0.0833\left[1 + 6.3244 + 1.6 + 0.5\right]$$

$$= 0.0833\left[9.4244\right]$$

$$= 0.78505$$

iii) Simpson's $\frac{3}{8}$ rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8}\left[y_0 + 3(y_1+y_2) + 2(y_3) + y_n\right]$$

$$= \frac{3(0.25)}{8}\left[1 + 3(0.9411 + 0.8) + 2(0.64) + 0.5\right]$$

$$= 0.09375\left[1 + 3(1.7411) + 2(0.64) + 0.5\right]$$

$$= 0.09375\left[1 + 5.2233 + 1.28 + 0.5\right]$$

$$= 0.09375\left[8.0033\right]$$

$$= 0.750309$$

2. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using the three rules $h = 0.5$

Sol:

Here $h = 0.5$

x	0	0.5	1
$y = \frac{1}{1+x^2}$	1	0.8	0.5
	y_0	y_1	y_2

i) Trapezoidal rule

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[y_0 + 2(y_1) + y_n\right]$$

$$= \frac{0.5}{2}\left[1 + 2(0.8) + 0.5\right]$$

$$= 0.25\left[1 + 1.6 + 0.5\right]$$

$$= 0.25\left[3.1\right]$$

$$= 0.775$$

ii) Simpson's $\frac{1}{3}$ rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[y_0 + 4(y_1) + y_n\right]$$

$$= \frac{0.5}{3}\left[1 + 4(0.8) + 0.5\right]$$

$$= 0.1666\left[1 + 3.2 + 0.5\right]$$

$$= 0.1666\left[4.7\right]$$

$$= 0.78302$$

ii) Simpson's $\frac{3}{8}$ rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8}\left[y_0 + 3(y_1) + y_n\right]$$

$$= \frac{(0.5)3}{8}\left[1 + 3(0.8) + 0.5\right]$$

$$= 0.1875\left[1 + 2.4 + 0.5\right]$$

$$= 0.1875\left[3.9\right]$$

$$= 0.73125$$

3. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using the three rules $h = 0.125$

Sol:

Here $h = 0.125$

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1
$y = \frac{1}{1+x^2}$	1	0.9846	0.9411	0.8767	0.8	0.7191	0.64	0.5663	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_n

i) Trapezoidal rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7) + y_n\right]$$

$$= \frac{0.125}{2}\left[1 + 2(0.9846 + 0.9411 + 0.8767 + 0.8 + 0.7191 + 0.64 + 0.5663) + 0.5\right]$$

$$= 0.0625\left[1 + 2(5.5278) + 0.5\right]$$

$$= 0.0625\left[1 + 11.0556 + 0.5\right]$$

$$= 0.0625\left[12.5556\right]$$

$$= 0.784725$$

ii) Simpson's $\frac{1}{3}$ rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6) + y_n\right]$$

$$= \frac{0.125}{3}\left[1 + 4(0.9846 + 0.8767 + 0.7191 + 0.5663) + 2(0.9411 + 0.8 + 0.64) + 0.5\right]$$

$$= 0.0416\left[1 + 4(3.1467) + 2(2.3811) + 0.5\right]$$

$$= 0.0416\left[1 + 12.5868 + 4.7622 + 0.5\right]$$

$$= 0.0416\left[18.849\right]$$

$$= 0.7841184$$

iii) Simpson's $\frac{3}{8}$ Rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2 + y_4 + y_5 + y_7) + 2(y_3 + y_6) + y_n\right]$$

$$= \frac{3(0.125)}{8}\left[1 + 3(0.9846 + 0.9411 + 0.8 + 0.7191 + 0.5663) + 2(0.8767 + 0.64) + 0.5\right]$$

$$= \frac{0.375}{8}\left[1 + 3(4.0111) + 2(1.5167)\right]$$

$$= 0.0468\left[1 + 12.0333 + 3.0334 + 0.5\right]$$

$$= 0.0468\left[16.5667\right]$$

$$= 0.77532$$

13. Evaluate $\int_0^{\pi/2} \sin x \, dx$ by dividing the range of integration into four equal parts using i) Trapezoidal rule ii) Simpson's $1/3$ rule iii) Simpson's $3/8$ rule

Sol

Here the length of the interval is

$$h = \frac{\frac{\pi}{2} - 0}{4} = \frac{\pi}{2} \times \frac{1}{4} = \frac{\pi}{8}$$

x	0	$\pi/8$	$2\pi/8$	$3\pi/8$	$\pi/2$
$y = \sin x$	0	0.3826	0.7071	0.9238	1
	y_0	y_1	y_2	y_3	y_n

i) Trapezoidal rule

$$\int_0^{\pi/2} \sin x \, dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_n\right]$$

$$= \frac{\pi/8}{2}\left[0 + 2(0.3826 + 0.7071 + 0.9238) + 1\right]$$

$$= \frac{\pi}{8} \times \frac{1}{2}\left[0 + 4.027 + 1\right]$$

$$= \frac{\pi}{16}\left[5.027\right]$$

$$= 0.987049$$

$$= 0.9871$$

ii) Simpson's $\frac{1}{3}$ rule

$$\int_0^{\pi/2} \sin x \, dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3) + 2(y_2) + y_n\right]$$

$$= \frac{\pi/8}{3}\left[0 + 4(0.3826 + 0.9238) + 2(0.7071) + 1\right]$$

$$= \frac{\pi}{24}\left[0 + 5.2256 + 1.4142 + 1\right]$$

$$= \pi/24 \left[7.6398\right]$$

$$= 1.00004748$$

1.00004

iii) Simpson's $\frac{3}{8}$ rule

$$\int_0^{\pi/2} \sin x \, dx = \frac{3h}{8}\left[y_0 + 3(y_1 + y_2) + 2(y_3) + y_n\right]$$

$$= \frac{3(\pi/8)}{8}\left[0 + 3(0.3826 + 0.7071) + 2(0.9238) + 1\right]$$

$$= \frac{3(\pi/8)}{8}\left[0 + 3.2691 + 1.8476 + 1\right]$$

$$= 3\pi/8 \times \frac{1}{8}\left[6.1167\right]$$

$$= \frac{3\pi}{64}\left[6.1167\right]$$

$$= 0.90076$$

14. The velocity (v) of the particle at distance (s) from the point on its path is given by the table

t	0	10	20	30	40	50	60	feet
v	47	58	64	65	61	52	38	feet/sec

estimate the time taken to travel 60 feet by using Simpson's $1/3$ rule. Compare the result with Simpson's $3/8$ rule

Sol

We know that the rate of change of displacement is velocity

$$v = \frac{ds}{dt} \quad (\text{or}) \quad v\,dt = ds$$

$$dt = \frac{ds}{v} \longrightarrow (1)$$

Here we want to find the time taken to travel 60 feet therefore equation (1) from 0 to

we got $\int_0^{60} dt = \int_0^{60} \frac{1}{v} \, ds$

time to taken to travel 60 feet is

$$t = \int_0^{60} \frac{1}{v}\,ds = \int_0^{60} y\,dx$$

The given table is

$x = s$	0	10	20	30	40	50	60	feet
$y = \frac{1}{v}$	0.02127	0.01723	0.01863	0.01838	0.01640	0.01923	0.0263	feet/sec
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	

Here, $h = 10$

i) Simpson's $\frac{1}{3}$ rule:

$$\int_0^{60} y\,dx = \frac{h}{3}\left[y_0 + 4(y_1+y_3+y_5) + 2(y_2+y_4) + y_n\right]$$

$$= \frac{10}{3}\left[0.02127 + 4(0.01723 + 0.01838 + 0.01923) + 2(0.01863 + 0.01640) + 0.0263\right]$$

$$= 3.3333\left[0.02127 + 4(0.05184) + 2(0.03203) + 0.0263\right]$$

$$= 3.3333\left[0.02127 + 0.20736 + 0.06406 + 0.0263\right]$$

$$= 3.3333\left[0.31899\right]$$

$= 1.06328$ Seconds

ii) Simpson's $\frac{3}{8}$ rule:

$$\int_0^{60} y\,dx = \frac{3h}{8}\left[y_0 + 3(y_1+y_2+y_4+y_5) + 2(y_3) + y_n\right]$$

$$= \frac{3(10)}{8}\left[0.02127 + 3(0.01723 + 0.01863 + 0.01640 + 0.01923) + 2(0.01838) + 0.0263\right]$$

$$= \frac{30}{8}\left[0.02127 + 3(0.06849) + 2(0.01838) + 0.0263\right]$$

$$= 3.75\left[0.02127 + 0.20547 + 0.03676 + 0.0263\right]$$

$$= 3.75\left[0.2898\right]$$

$= 1.06425$ Seconds