Integration by parts

We know that if u, v are functions of x, y

then $\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ (product rule)

Integrating both sides with respect to 'x'

$$\int \frac{d}{dx}(uv) = \int u\frac{dv}{dx}\,dx + \int v.\frac{du}{dx}\,dx$$

$$uv = \int u\,dv + \int v\,du$$

$$\int u\,dv = uv - \int v\,du.$$

ETALI ← u

E - Exponential
T - Trigonometric
A - Algebraic
L - Log.
I - Inverse,

1) Integral $xe^x\,dx$

soln:-

$u = x \qquad \int dv = \int e^x\,dx$

$du = dx \qquad v = e^x$

$$\int u\,dv = uv - \int v.du$$

$$= xe^x - \int e^x\,dx$$

$$= xe^x - e^x$$

$$= e^x(x-1)$$

2) $\int x \sin x\,dx$

sol:-

$u = x \quad ; \quad \int dv = \int \sin x\,dx$

$du = dx \quad ; \quad v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$= -x\cos x - \int -\cos x\,dx$

$= -x\cos x + \sin x$

$\int u\,dv = \sin x - x\cos x.$

3) $\int \log x\,dx$

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$u = \log x$; $\int dv = \int dx$

$v = x$

$du = \frac{1}{x}\,dx$

$\int u.dv = uv - \int v.du$

$= \log x \cdot x - \int x \cdot \frac{1}{x}\,dx$

$= \log x \cdot x - x$

$= x(\log x - 1)$

4) $\int x \sin^{-1} x\,dx.$

$u = \sin^{-1} x$ ~~$\int dv = \int dx \cdot x$~~

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$; $\int \frac{dv}{dx} = x$

$v = x.dx$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$; $v = \frac{x^2}{2}$

$\int u\,dv = uv - \int v.du$

$= \sin^{-1}x\,\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2}\frac{1}{\sqrt{1-x^2}}\,dx$

put $x = \sin\theta$

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$$\frac{dx}{d\theta} = \cos\theta \Rightarrow dx = \cos\theta \, d\theta$$

$$\boxed{dx = \cos\theta \, d\theta}$$

$$\int u \cdot dv = \frac{x^2}{2}\sin^{-1}x - \int \frac{\sin^2\theta}{2} \cdot \frac{1}{\sqrt{1-\sin^2\theta}} \cos\theta \, d\theta$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{\sin^2\theta \cos\theta \, d\theta}{\sqrt{\cos^2\theta}}$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \sin^2\theta \, d\theta$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{1}{2}(1-\cos 2\theta)\, d\theta$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{4}\int d\theta - \int \cos 2\theta \, d\theta$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{4}\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\theta^2 + \frac{\sin 2\theta}{2}$$

5) $\int e^x\left(\frac{1+\sin x}{1+\cos x}\right) dx$

$\sin x = 2 \frac{\sin x}{2} \frac{\cos x}{2}$

$1+\cos x = 2\cos^2\frac{x}{2}$

$$= \int e^x \left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) dx$$

$$= \int e^x \left(\frac{1}{2\cos^2\frac{x}{2}} + \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right) dx$$

$$= \int \frac{e^x}{2} \sec^2\frac{x}{2}\, dx + \int e^x \tan\frac{x}{2}\, dx$$

$$= \frac{1}{2}\int e^x \sec^2\frac{x}{2} + \int e^x \tan\frac{x}{2}\, dx$$

$$\therefore \sec^2\frac{x}{2} = d\left(2\tan\frac{x}{2}\right)$$

$$= \frac{1}{2}\int e^x\, 2\tan\frac{x}{2}\, dx + \int e^x \tan\frac{x}{2}\, dx$$

$u = e^x \quad ; \quad \int dv = \sec^2\frac{x}{2}$

$du = e^x\, dx \quad ; \quad v = 2\tan\frac{x}{2}$

$$\int u.dv = uv - \int v.du$$

$$= \frac{1}{2}\left[e^x(2)\tan\frac{x}{2} - \int 2\tan\frac{x}{2} e^x dx + \int e^x \tan\frac{x}{2}\right]$$

$$= e^x \tan\frac{x}{2}.$$

6) $\int \frac{\log x}{(1+x)^2}\, dx.$

Sol:-

$$\int \frac{\log x}{(1+x)^2}\, dx = \int \log x\; d\left(\frac{-1}{1+x}\right)$$

$u = \log x \qquad v = \frac{-1}{1+x}$

$du = \frac{1}{x}\, dx$

$$\int u.dv = uv - \int v.du$$

$$= \log x\left(\frac{-1}{1+x}\right) - \int\left(\frac{-1}{1+x}\right)\frac{1}{x}\, dx.$$

$$= \log x\left(\frac{-1}{1+x}\right) + \int\left(\frac{1}{x} - \frac{1}{1+x}\right) dx$$

$$= \frac{-\log}{1+x} + \log x - \log(1+x)$$