LINEARLY DEPENDANT AND INDEPENDENT VECTORS

VECTOR SPACES

- **Definition:** A vector space *V* is a set that is closed under finite vector addition and scalar multiplication.

- An operation called vector addition that takes two vectors v, w ∈ V , and produces a third vector, written v + w ∈ V .

- An operation called scalar multiplication that takes a scalar c ∈ F and a vector v ∈ V , and produces a new vector, written cv ∈ V.

LINEAR INDEPENDENCE VECTORS

- **Definition:** An indexed set of vectors $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in is said to be **linearly independent** if the vector equation

$$x_1 v_1 + x_2 v_2 + \ldots + x_p v_p = 0$$

has only the trivial solution. The set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is said to be **linearly dependent** if there exist weights $c_1, \ldots, c_p$, not all zero, such that,

$$c_1 v_1 + c_2 v_2 + \ldots + c_p v_p = 0 \quad \text{----(1)}$$

LINEAR INDEPENDENCE VECTORS

- Equation (1) is called a **linear dependence relation** among $\mathbf{v}_1, \ldots, \mathbf{v}_p$ when the weights are not all zero.
- An indexed set is linearly dependent if and only if it is not linearly independent.
- **Example 1:** Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

LINEAR DEPENDENCE VECTORS

- **Definition:** A finite set ***S*** = {x1, x2, . . . , xm} of vectors in ***R***n is said to be linearly dependent if there exist scalars (real numbers) c1, c2, . . . , cm, not all of which are 0, such that

$$c_1x_1 + c_2x_2 + \ldots + c_mx_m = 0.$$

LINEAR DEPENDENCE VECTORS

- Any set containing the vector 0 is linearly dependent, because for any c 6= 0, c0 = 0. 3.

- In the definition, we require that not all of the scalars c1, . . . , cn are 0. The reason for this is that otherwise, any set of vectors would be linearly dependent.

- **Example 1:** Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$.

MATHEMATICAL PROBLEMS

Question: S= {(1,-1,3),(1,4,5),(2,-3,-7)} ⊂ R^3. S is LD or LI.

Solution: We name the vectors as,

(1,-1,3) = v1 ; (1,4,5) = v2 and (2,-3,-7) = v3

We can construct a matrix by using the vectors,

$$\begin{pmatrix} 1 & -1 & 3 \\ 1 & 4 & 5 \\ 2 & -3 & -7 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 3 \\ 0 & -5 & -2 \\ 0 & 0 & 63 \end{pmatrix}$$

(REF)

Since there is no zero row in REF the vectors in S are Linear Independent.

MATHEMATICAL PROBLEMS

Question: S= {(2,1,1),(3,-4,6),(4,-9,11)} ⊂ $\boldsymbol{R}^3$. S is LD or LI.

Solution: We name the vectors as,

(2,1,1) = v_1 ; (3,-4,6) = v_2 and (4,-9,11) = v_3

We can construct a matrix by using the vectors,

$$\begin{pmatrix} 2 & 1 & 1 \\ 3 & -4 & 6 \\ 4 & -9 & 11 \end{pmatrix} \sim \begin{pmatrix} 2 & 1 & 1 \\ 0 & 11 & -9 \\ 0 & 0 & 0 \end{pmatrix}$$

(REF)

Since there is a zero row in REF the vectors in S are Linear Dependent.

Laplace Equation in Three Coordinate System

- **In Cartesian coordinates:**

$$\nabla V = \frac{\partial V}{\partial x}\overline{a_x} + \frac{\partial V}{\partial y}\overline{a_y} + \frac{\partial V}{\partial z}\overline{a_z}$$

and,

$$\nabla A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Knowing

$$\nabla^2 V = \nabla \cdot \nabla V$$

Hence, Laplace's equation is,

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

- **In cylindrical coordinates:**

$$\nabla \cdot V = \frac{\partial V}{\partial \rho}\overline{a_\rho} + \frac{1}{\rho}\frac{\partial V}{\partial \emptyset}\overline{a_\emptyset} + \frac{\partial V}{\partial z}\overline{a_z}$$

and,

$$\nabla \cdot A = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\emptyset}{\partial \emptyset} + \frac{\partial A_z}{\partial z}$$

Knowing

$$\nabla^2 V = \nabla \cdot \nabla V$$

Hence, Laplace's equation is,

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\frac{\partial V}{\partial \rho}) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \emptyset^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

- **In spherical coordinates:**

$$\nabla \cdot V = \frac{\partial V}{\partial r}\overline{a_r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\overline{a_\theta} + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \emptyset}\overline{a_\emptyset}$$

and,

$$\nabla \cdot \bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\emptyset}{\partial \emptyset}$$

Knowing

$$\nabla^2 V = \nabla \cdot \nabla V$$

Hence, Laplace's equation is,

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial V}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial V}{\partial \theta}) + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial \emptyset^2} = 0$$