Applied Physics - I

Unit - V ⇒ Alternating Current

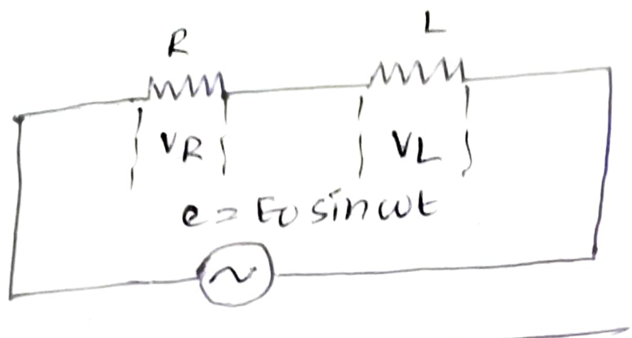
Alternating Current

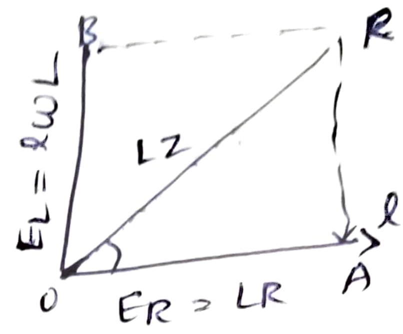
A.C circuits with Double Components :-

i) A.C circuit having inductance and resistance

An alternating e.m.f $\boxed{e = E_0 \sin \omega t}$ is applied across a circuit containing a resistance R and an inductance L. If i is the instantaneous current, the p.d across R is $\boxed{E_R = Ri}$ and the p.d across the inductance is $\boxed{E_L = i\omega L}$

The p.d across the resistance is in phase with the current through the resistance. The p.d across the inductance leads the current through the inductance by an angle $\pi/2$. The current in the circuit i the same at any point. Hence current is taken as the reference axis. A vector voltage diagram is drawn. Where the vector OA represents the magnitude and directions of the p.d across the resistance. The vector OB represents the p.d across the inductance both in magnitude and direction. The resultant vector E will represent the effective p.d across the inductance and the resistance





$$OC = \sqrt{OA^2+OB^2}$$

$$= \sqrt{E_R^2+E_L^2}$$

$$= i\sqrt{R^2+(L\omega)^2}$$

$$\boxed{i = \sqrt{R^2+(L\omega)^2}} \Rightarrow (1)$$

The term $\sqrt{R^2+(L\omega)^2}$ is called impedance of the circuit and measured in the unit of ohm. It is denoted by the letter Z.

$$Z = e/i$$

$$= \sqrt{R^2+(L\omega)^2} \Rightarrow (2)$$

From the vector diagram

$$\tan\theta = AC/OA = \omega L/R \Rightarrow (3)$$

tan θ is positive. This shows that the current lags behind the applied e.m.f by a phase angle θ where,

$$\theta = \tan^{-1}(\omega L/R) \Rightarrow (4)$$

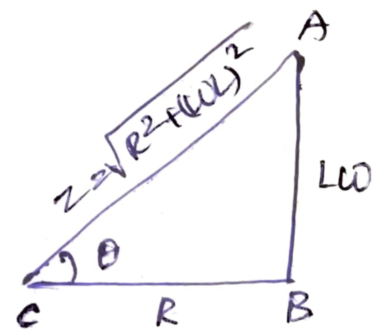
Hence the current at any instant of time is,

$$i = \frac{e_0}{\sqrt{R^2+(L\omega)^2}} \sin(\omega t+\theta) \Rightarrow (5)$$

The peak value of current,

$$i_0 = \frac{e}{\sqrt{R^2+(\omega L)^2}} \sin(\omega t+\theta) \Rightarrow (6)$$

$$\boxed{i = i_0 \sin(\omega t+\theta)} \Rightarrow (7)$$



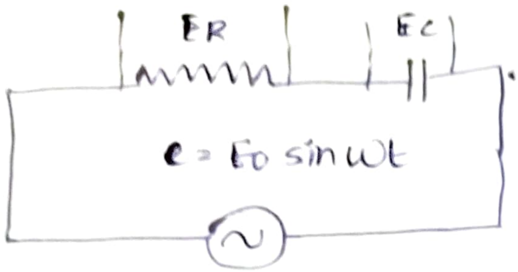
A triangle whose sides are proportional to R, ωL and z is called the impedance triangle. The impedance triangles is as shown in fig. The hypotenuse will represent the impedance of the circuit and ∠ACB gives the phase lag of the current behind the voltage.

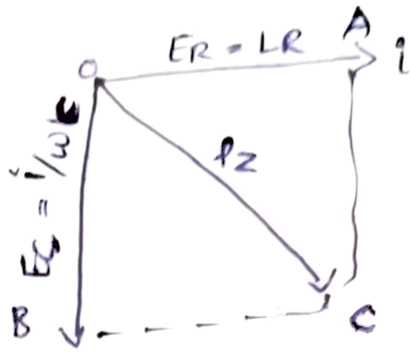
The reciprocal of the impedance is called the admittance of the circuit and is denoted by the letter Y.

$$Y = \frac{1}{Z} = \frac{e}{\sqrt{(R)^2+(L\omega)^2}} \text{ mho} \Rightarrow (8)$$

b) A.C circuit having resistance and capacitance

An alternating e.m.f $\boxed{e = E_0 \sin \omega t}$ is applied across a resistance R and a condenser of capacity C connected in series. If i is the instantaneous current, the p.d across the resistance ... and the p.d across the capacitor is $\boxed{E_C = -i/\omega C}$





The p.d across the resistance is in phase with the current through the resistance. The p.d across the inductance leads the current through the inductance by an angle $\pi/2$. The current in the circuit is the same at any point. Hence current is taken as the reference axis. A vector voltage diagram is drawn. Where the vector OA represents the p.d across the resistance both in magnitude and direction. The vector OB represents the p.d across the capacitor both in magnitude and direction. The resultant vector OC represents the effective p.d across the capacitor and the resistance in series. From the voltage diagram,

$$OC = \sqrt{OA^2 + OB^2}$$

$$e = \sqrt{E_R^2 + E_C^2}$$

$$= i\sqrt{R^2 + (1/\omega c)^2}$$

$$i = \frac{e}{\sqrt{R^2 + (1/C\omega)^2}} \rightarrow ①$$

The term $\sqrt{R^2 + (1/C\omega)^2}$ is called impedance of the circuit.

Impedance $z = \frac{e}{i}$

$$= \sqrt{R^2 + (1/C\omega)^2} \rightarrow ②$$

From the vector diagram,

$\tan\theta = AC/OA$

$$= -1/(C\omega)^2 \rightarrow ③$$

$\tan\theta$ is negative : This shows that the current lead the applied e.m.f by a phase angle θ where,

$$\theta = \tan^{-1}(1/C\omega)^2 \rightarrow ④$$

Hence the current at any instant of time is,

$$i = \frac{e_0}{\sqrt{R^2 + (1 + C\omega)^2}} \sin(\omega t + \theta) \rightarrow ⑤$$

The peak value of current,

$$i_0 = \frac{e}{\sqrt{R^2 + (1/C\omega)^2}} \rightarrow ⑥$$

$$i = i_0 \sin(\omega t + \theta) \rightarrow ⑦$$

The admittance of the circuit is,

$$Y = \frac{1}{z} = \frac{e}{\sqrt{R^2 + (1/C\omega)^2}} \text{ mho} \rightarrow ⑧$$

Power in an a.c circuit

Power is defined as the rate of doing work. In the d.c circuit power is the product of current and voltage. If the current is in ampere and voltage in volt, then the power is expressed in watt. In the case of a.c circuit, e and i vary continuously. So in a.c circuit, we have to calculate the work done at any instant of time and then the power for the complete cycle is calculated.

a) Power in a pure Resistive circuit

Let an alternating e.m.f be applied to a circuit containing only R. At any instant of time let $\boxed{e = E_0 \sin \omega t}$ be the instantaneous value of the applied voltage and $\boxed{i = I_0 \sin \omega t}$ be the instantaneous current.

Power at that instant $= e\,i$

$$= E_0 I_0 \sin^2 \omega t \Rightarrow (1)$$

The average power dissipated during a complete cycle is,

$$P = \frac{1}{T} \int_0^T E_0 I_0 \, 2 \sin^2 \omega t \, dt$$

$$P = \frac{E_0 I_0}{2T} \int_0^T 2\sin^2 \omega t \, dt$$

$$P = \frac{E_0 I_0}{2T} \int_0^T (1-\cos 2\omega t)\, dt$$

$$= \frac{E_0 I_0}{2}$$

$$\boxed{P = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}}}$$

But we know that $E_0/\sqrt{2}$ is the r.m.s value of voltage and $I_0/\sqrt{2}$ is the r.m.s value of current.

Average power $P = E_{r.m.s} I_{r.m.s}$

b) power in inductive current

Let an alternating e.m.f be applied to a circuit containing only inductance L. The current through a pure inductance lags behind the applied e.m.f in phase by $\pi/2$

$$e = E_0 \sin \omega t$$

$$I = I_0 \sin(\omega t - \pi/2)$$

Instantaneous power $= ei$

$$= E_0 I_0 \sin \omega t - \sin(\omega t - \pi/2)$$

$$= -E_0 I_0 \sin \omega t - \cos \omega t$$

$$= -\frac{E_0 I_0}{2} \sin 2\omega t$$

Average power over one cycle is,

$$P = \frac{1}{T} \int_0^T \frac{E_0 I_0}{2} \sin 2\omega t - dt$$

$$= + \frac{E_0 I_0}{2T} \int_0^T \sin 2\omega t - dt$$

But $\int_0^T \sin 2\omega t - dt = 0$

Average power $P = 0$

Thus power dissipation in a pure inductance is zero and the current in such a circuit is known as wattless current. The choke coil work on this principle.

c) Power in capacitive current circuit

Let an alternating e.m.f be applied to circuit containing only capacitance C. The current through a pure capacitor leads the applied e.m.f in phase by $\pi/2$

$$e = E_0 \sin \omega t$$

$$i = I_0 \sin (\omega t + \pi/2)$$

Instantaneous Power $= ei$

$$= E_0 I_0 \sin \omega t - \sin (\omega t + \pi/2)$$

$$= E_0 I_0 \sin \omega t - \cos \omega t$$

$$= \frac{E_0 I_0}{2} \sin 2\omega t$$

As the average of $\sin 2\omega t$ over a cycle is zero. Hence the average power is zero. Thus the average power dissipation in a pure capacitance circuit is zero. Hence the current through it also wattless (or) idle.

d) Power in a circuit containing L and R in series :-

Let an alternating e.m.f be applied to a circuit containing an inductance L and a resistance R. At any instant of time let $\boxed{e = E_0 \sin\omega t}$ and $i = I_0 \sin(\omega t - \theta)$ where θ is the phase lag of the current behind the applied e.m.f

Instantaneous Power $= ei$

$$= E_0 I_0 \sin\omega t \cdot \sin(\omega t - \theta)$$

$$= E_0 I_0 \sin\omega t \left[\sin\omega t \cos\theta - \cos\omega t \cdot \sin\theta\right]$$

$$= E_0 I_0 \left[\sin^2\omega t \cdot \cos\theta - \sin\omega t \cdot \cos\omega t \cdot \sin\theta\right]$$

$$= E_0 I_0 \sin^2\omega t \cdot \cos\theta - \frac{E_0 I_0}{2} \cdot \sin\omega t \cdot \sin\theta$$

Average power in a complete cycle is,

$$P = \frac{1}{T}\int_0^T E_0 I_0 \cos\theta \cdot \sin^2\omega t \cdot dt$$

$$- \frac{1}{2}\int_0^T E_0 I_0 \sin\theta \cdot \sin 2\omega t \cdot dt$$

$$= \frac{E_0 I_0 \cos\theta}{T}\int_0^T \sin^2 \omega t - dt - \frac{E_0 I_0 \cos\theta}{2T}\int_0^T \sin 2\omega t - dt$$

But, $\int_0^T \sin^2 \omega t - dt = 1/2$

$$\int_0^T \sin 2\omega t - dt = 0$$

$$\text{Power } P = \frac{E_0 I_0}{2} - \cos\theta$$

$$= \frac{E_0 I_0}{2} \cos\theta$$

$$P = \frac{E_0}{\sqrt{2}} - \frac{I_0}{\sqrt{2}} \cos\theta$$

$$\boxed{P = E_{r.m.s}\, I_{r.m.s} \cos\theta}$$

It is clear that power depends that ~~only~~ one only on the r.m.s value of voltage and current but also on the cosine of the angle of lag between the current and voltage. Hence $\cos\theta$ is known as the power factor and

$$\theta = \tan^{-1}(L\omega/R)$$

$$\text{power factor } \cos\theta = \frac{R}{\sqrt{R^2 + (L\omega)^2}}$$

$$\text{Power factor} = \frac{\text{Resistance}}{\text{Impedance}}$$

$$= \frac{R}{Z}$$

⇒ The product $E_{r.m.s} I_{r.m.s}$ is known as apparent power.

$$\text{Power factor} = \frac{\text{The power}}{\text{Apparent power}}$$

True power = power factor × Apparent power

⇒ In a similar way, we can show that the power in a circuit containing R and c is,

$E_{r.m.s} \cdot I_{r.m.s} \cos\theta$

$$\text{Power factor } \cos\theta = \frac{R}{\sqrt{R^2+(1/c\omega)^2}}$$

e. <u>Wattless current:-</u>

If an a.c circuit is purely Inductive (or) purely capacitive, the phase angle is $\pi/2$. Hence $\cos\theta = 0$. Therefore the power consumed in such a circuit is zero. The current in such a circuit does not perform any useful work. so it is called wattless (or) Idle current.

f. <u>choke:-</u>

⇒ For many purpose, it is required to reduce the current in a given circuit with a minimum waste of power.

⇒ In an a.c circuit, when a resistance is used, the wastage of energy is FR. In order the avoid any wastage of energy, an inductance coil is used.

⇒ A choke consists of a coil of several turns of insulated wire of low resistance, but large inductance, wound over a laminated core. The core is layered and is made up of thin sheets of stalloy to reduce hysteresis losses.

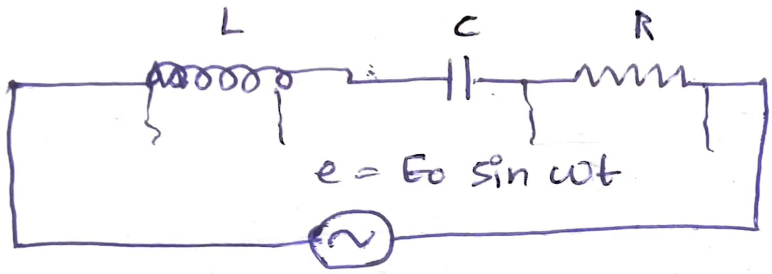
⇒ choke coils which are used on low frequency current have iron core. These are known as low frequency (or) audio frequency chokes.

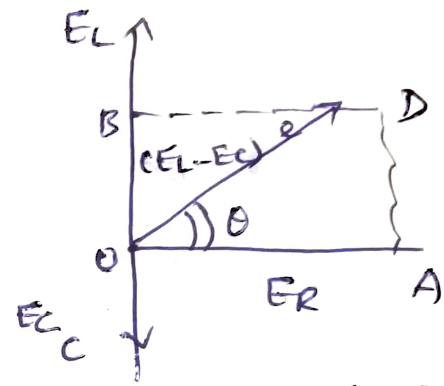
⇒ choke coils which are used on high frequency have air core. These are known as high frequency (or) radio frequency chokes.

A.C circuits having L, C and R

⇒ An alternating e.m.f $\boxed{e = -E_0 \sin \omega t}$ is applied to a circuit containing a resistance R, inductance L and a capacitance C. If i is the instantaneous current, the P.d across R is $E_R = RI$, the P.d across L is $E_L = i \cdot X_L$ and the P.d across C is

$$\boxed{E_C = iX_L}$$





⇒ The voltage drop across the resistance is in phase with the current, the voltage drop across the inductance is leading the current by a phase angle $\pi/2$ and the voltage drop across the capacitance is lagging the current by a phase angle $\pi/2$

⇒ The current in each of the component is the same. Hence the current is taken as the reference axis.

⇒ $(E_L - E_C)$ since the vectors are oppositely drawn. It is represented by OB in the fi

The diagonal OD represents the P.d across the whole circuit.

$$e^2 = E_R^2 + (E_L - E_C)^2$$

$$e = \sqrt{i^2R^2 + i^2(X_L - X_C)^2}$$

$$e = i\sqrt{R^2 + (X_L - X_C)^2}$$

$$i = \frac{e}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Impedance $Z = \frac{e}{I}$

$$= \sqrt{R^2 + (X_L - X_C)^2}$$

The phase angle θ is given by,

$$\tan\theta = \frac{AO}{OA} = \frac{E_L - E_C}{E_R}$$

$$\tan\theta = \frac{X_L - X_C}{R}$$

$(X_L - X_C)$ (or) $\left(L\omega - \frac{1}{C\omega}\right)$

Gives the phase relation between the current lag behind e.m.f. The current leads (or) lags the e.m.f depending upon the magnitude of $L\omega$ and $1/C\omega$

Case - I $L\omega > 1/C\omega$

$$\theta = \tan^{-1}\left[\frac{L\omega - (1/C\omega)}{R}\right]$$

$$\boxed{i = i_0 \sin(\omega t - \theta)}$$

Case - 2 $\quad L\omega < 1/C\omega$

$$\theta = \tan^{-1}\left[\frac{(1/C\omega) - L\omega}{R}\right]$$

$$\boxed{i = i_0 \sin(\omega t + \theta)}$$

a. Series Resonance circuits

When an alternating e.m.f is applied to a circuit containing L, C and R in series, the peak value of current is given by,

$$i_0 = \frac{e}{\sqrt{R^2 + (L\omega - 1/C\omega)^2}}$$

The peak value of current is e/R. When R is called small, the current becomes very large,

$$L\omega = 1/C\omega$$

$$\boxed{\omega^2 = 1/LC} \qquad \omega = \sqrt{1/LC}$$

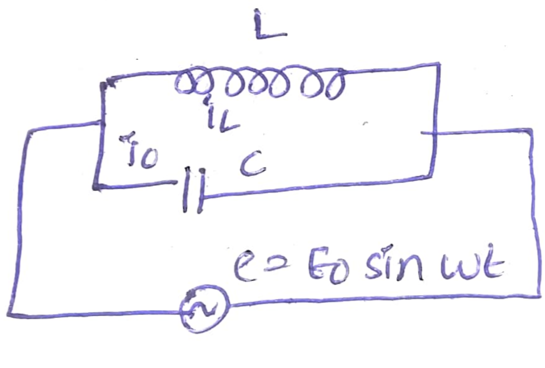
But frequency, $f = \omega/2\pi$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

⇒ Hence the frequency is called resonant frequency. The condition for resonance is that the frequency of the circuit. When the frequency approaches the resonant frequency f_r, the current begins to increase and reaches maximum at f_r.

b. Parallel Resonance Circuit

An inductance L of negligible resistance and a capacitor C are connected in parallel to an a.c supply. Let i_L be the current in the inductance.



$$i_L = \frac{E_0}{wL} \text{ and } i_C = \frac{E_0}{1/wC} = E_0 wC$$

The total current in the circuit will be the vector sum of the two currents.

$$i = E_0 \left[\frac{1}{wL} - wC \right]$$

At resonance,

$$\boxed{i_L = i_C}$$

But frequency

$$\boxed{f = \frac{\omega}{2\pi}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Thus the parallel resonant frequency. is the same as the frequency of the series resonant circuit.

Thus the current through the parallel resonant circuit is zero. Here this circu is called rejector circuit.