Applied Physics - I

Unit - 3: Current Electricity

Laplace's Law:

Using a Vibration magnetometer Biot & Savart studied intensity of the magnetic field due to a long vertical conductor carrying current. From their experiment, they found out the following facts, the intensity of the fields varies,

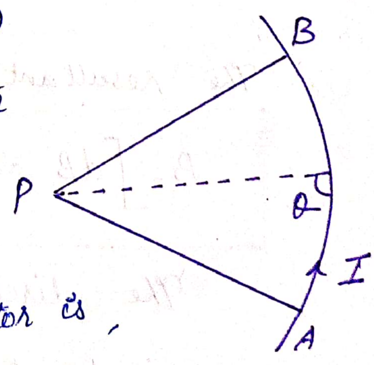
(i) directly as the strength of the current through the conductor

(ii) inversely as the perpendicular distance of the point from the conductor.

This is known as Biot and Savart's law.

Laplace and Ampere showed that the results obtained by Biot and Savart can be mathematically formulated. This is referred as Laplace or Ampere's law.

Let AB be a conductor through which a current of I ampere. (fig)



According to laplace, the intensity of the magnetic field at any point P due to a small element of the conductor is,

(i) $dB \propto i$

directly proportional to the strength of the current (i)

(ii) directly proportional to the sine of the angle θ

$dB \propto \sin\theta$

(iii) directly proportional to the length of the element (dl)

$dB \propto dl$

(iv) inversely proportional to the square of the distance (r)

$dB \propto \frac{1}{r^2}$ $\quad \therefore OP = r$

The magnetic intensity at P,

$$dB \propto \frac{i\,dl\sin\theta}{r^2} \quad \text{--- (1)}$$

$$\boxed{dB = \frac{k\,i\,dl\sin\theta}{r^2}} \quad \text{--- (2)}$$

where k is a constant. This is called Ampere's law.

* k depends upon the property of the medium between the element and the point 'P'.

In SI units, $k = \frac{\mu_0}{4\pi} = 10^{-7}$ wb/A/m

where, $\mu_0 \rightarrow$ permeability of free space.

$\mu_0 = 4\pi \times 10^{-7}$ weber $A^{-1} m^{-1}$

Now, the equation (2) becomes,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i\,dl\sin\theta}{r^2} \quad \text{--- (3)}$$

$\therefore$ The resultant magnetic field at P,

$$\boxed{B = \int dB = \frac{\mu_0}{4\pi} \int \frac{i\,dl\sin\theta}{r^2}} \quad \text{--- (4)}$$

The direction of the magnetic field is perpendicular to the plane containing the element and the point P.

Magnetic field intensity due to long straight conductor:

Let xy be a straight conductor carrying a current of i in the direction y to x. Let 'p' be a point 'a' from xy at which the magnetic intensity is to be determined. Let AB be a small element of length 'dl' of the conductor. Join PA and PB and draw PO ⊥r to xy.

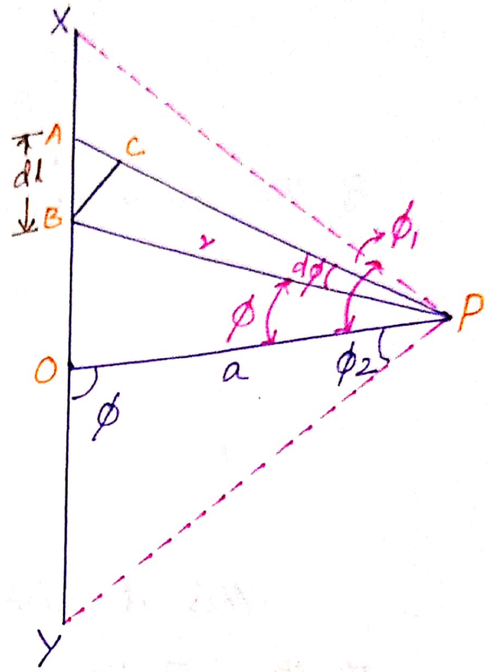


fig. Let $\angle OPB = \phi$,

$\angle O$

$\angle BPA = d\phi$,

Draw BC ⊥r to AP.

Let, $PO = a$, $BP = AP = r$

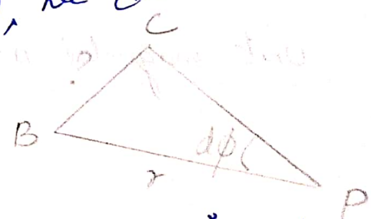
From, the Laplace law, the magnetic field at 'P',

$$dB = \frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2} \quad — ①$$

In ΔABC, $\sin\theta = \frac{BC}{AB}$

$\therefore AB.\sin\theta = BC$ (or) $BC = dl\sin\theta$ — ②

Substituting equ ② in equ ①, we get



$$dB = \frac{\mu_0}{4\pi}.\frac{iBC}{r^2} \quad — ③$$

From fig. $BC = rd\phi$ — ④

$$\therefore dB = \frac{\mu_0}{4\pi}.\frac{i\cancel{r}d\phi}{r^{\cancel{2}}} \quad (\text{or}) \quad dB = \frac{\mu_0}{4\pi}.\frac{i}{r}.d\phi \quad — ⑤$$

In ΔOPB,

$$\cos\phi = \frac{OP}{PB} = \frac{a}{r} \quad \therefore r = \frac{a}{\cos\phi} \quad — ⑥$$

Now, equ ⑤, becomes

$$dB = \frac{\mu_0}{4\pi}.\frac{i.\cos\phi}{a}.d\phi \quad — ⑦$$

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Join Px and Py. Let $\angle OPX = \phi_1$ and $\angle OPY = \phi_2$

The total intensity of the magnetic field,

limits $\phi = -\phi_1$ and $\phi = \phi_2$

$$B = \frac{\mu_0 i}{4\pi a} \int_{-\phi_1}^{\phi_2} \cos\phi \, d\phi = \frac{\mu_0 i}{4\pi a} \left[\sin\phi\right]_{-\phi_1}^{\phi_2}$$

$$= \frac{\mu_0 i}{4\pi a} \left[\sin\phi_2 - \sin(-\phi_1)\right]$$

$$\boxed{B = \frac{\mu_0 i}{4\pi a} \left[\sin\phi_2 + \sin\phi_1\right]} \quad \text{— (9)}$$

If the conductor is very long, $\phi_1 = 90°$, $\phi_2 = 90°$

$$B = \frac{\mu_0 i}{4\pi a} [1+1] = \frac{\mu_0 i}{\cancel{4}_{2}\pi a} \cdot \cancel{2}$$

$$\boxed{B = \frac{\mu_0 i}{2\pi a}} \quad Wb/m^2 \quad \text{— (10)}$$

This is Biot - Savart's law. The direction of the intensity is perpendicular to the plane of the paper and inwards.

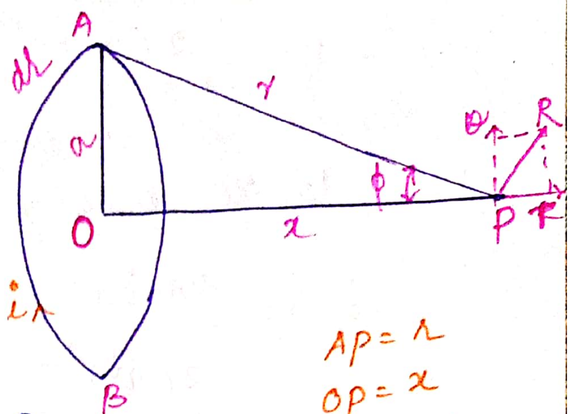
This is also the force experienced by a unit magnetic pole at a point.

Magnetic field intensity due to Circular Coil :

Consider a circular coil of radius 'a'

n = no. of turns.

* Small element = dl



AP = r
OP = x
∠OAP = ϕ

The field intensity at P,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i\,dl \sin\theta}{r^2} \longrightarrow ①$$

AP is ⊥r to the direction of the Current as at A.

Hence $\theta = 90^\circ$

$$\therefore \quad dB = \frac{\mu_0}{4\pi} \cdot \frac{i\,dl}{r^2} \longrightarrow ②$$

* Resolving into 2 components, PT and PQ

(i) $\frac{\mu_0}{4\pi} \times \frac{i\,dl}{r^2} \cdot \sin\phi$ along PT

(ii) $\frac{\mu_0}{4\pi} \times \frac{i\,dl}{r^2} \cos\phi$ along PQ

Hence, the total mag. field intensity at P is,

$$B = \int dB = \frac{\mu_0}{4\pi} \cdot \frac{i}{r^2} \int dl.\sin\phi = \frac{\mu_0}{4\pi} \frac{i \sin\phi}{r^2} \int dl.$$

from fig, $\sin\phi = \frac{a}{r}$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{i a}{r^3} \int dl =$$

$\int dl$ = circumference = $2\pi a$.

$\therefore$ there are 'n' no. of turns $\int dl = 2\pi n.a$

$$B = \frac{\mu_0}{\cancel{4\pi}\,2} \cdot \frac{i a}{r^3} \cdot \cancel{2\pi} n.a = \frac{\mu_0}{2} \cdot \frac{n i a^2}{r^3}$$

$$B = \frac{\mu_0}{2} \cdot \frac{n i a^2}{r^3}$$

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$$\therefore \boxed{B = \frac{\mu_0 n i a^2}{2(x^2+a^2)^{3/2}} \quad wb/m^2}$$

$r^2 = x^2+a^2$

$r^3 = (x^2+a^2)^{3/2}$

Magnetic field at the Centre of the Coil:

At the centre of the coil, $x=0$

$$\therefore B = \frac{\mu_0 n i a^2}{2(a^2)^{3/2}} = \frac{\mu_0 n i \cancel{a^2}}{2a^{\cancel{3}}} = \frac{\mu_0 n i}{2a}$$

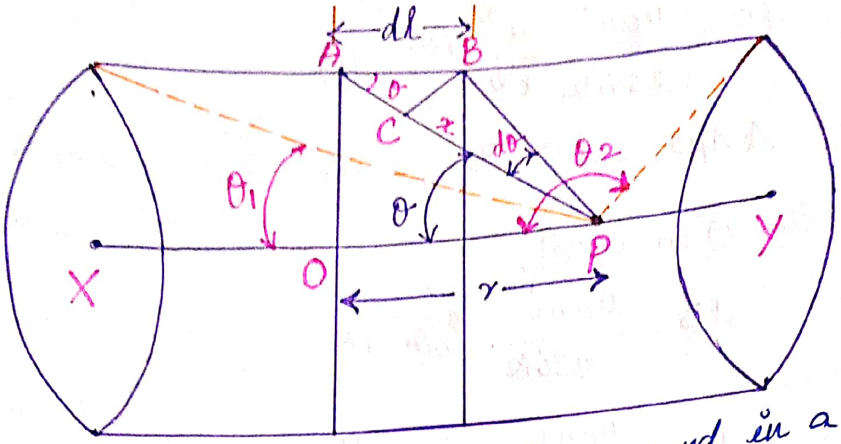
$$\boxed{B = \frac{\mu_0 n i}{2a}} \; wb/m^2$$

Variation of mag.field along the axis of the coil:

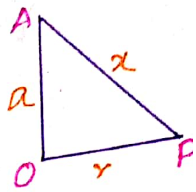
* At the Centre of the coil, the magnitude of field intensity is greater.
* The field intensity decreases as we go away from the Centre, but its rate of variation is not a Constant. The variation of 'B' with distance can be mathematically investigated.

Magnetic field intensity at a point on the axis of a Solenoid:

radius $= a$
length $= dl$



A Solenoid is a long wire wound in a closed-packed helix and carrying an electric current.



$xy \rightarrow$ axis of the solenoid

$n \rightarrow$ no. of turns per unit length.

$a \rightarrow$ radius.

Let us assume that the thickness of the wire is small compared to the radius of the Solenoid.

$i =$ Current flowing through the Solenoid

The Mag. field intensity at P,

$$dB = \frac{\mu_0 n i a^2 . dl}{2(a^2+x^2)^{3/2}} \longrightarrow ①$$

ΔAPO,

$a^2 + r^2 = x^2$

$$dB = \frac{\mu_0 n i a^2 . dl}{2(a^2+r^2)^{3/2}} = \frac{\mu_0 n i a^2 . dl}{2(x^2)^{3/2}}$$

$\because \sin\theta = \frac{BC}{AB}$

$\sin\theta = \frac{BC}{dl}$

$$dB = \frac{\mu_0 n i a^2}{2(x^3)} \rightarrow ②$$

$\boxed{BC = dl \sin\theta} \rightarrow ③$

But

$\boxed{BC = x\, d\theta} \rightarrow ④$

Compare eqn ③ & ④

$x\, d\theta = dl \sin\theta$

$$\boxed{dl = \frac{x . d\theta}{\sin\theta}} \longrightarrow ⑤$$

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Sub equ (5),

$$dB = \frac{\mu_0 n i a^2}{2x^{\cancel{3}}} \cdot \frac{\cancel{x}\,d\theta}{\sin\theta} = \frac{\mu_0 n i a^2\,d\theta}{2x^2 \sin\theta}$$

$$dB = \frac{\mu_0 n i}{2\sin\theta} \cdot \frac{a^2}{x^2} \cdot d\theta \quad \text{— (6)}$$

ΔAPO, $\angle APO = \theta$; $\sin\theta = \frac{a}{x}$; $\sin^2\theta = \frac{a^2}{x^2}$ $\rightarrow$ (7)

Sub (7) in equ (6),

$$dB = \frac{\mu_0 n i}{2\cancel{\sin\theta}} \cdot \sin^{\cancel{2}}\theta \cdot d\theta$$

$$dB = \frac{\mu_0 n i}{2} \cdot \sin\theta \cdot d\theta \quad \text{— (8)}$$

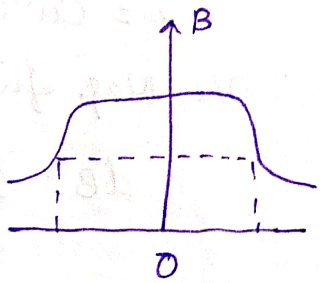
Total Mag. field intensity at P,

$$B = \frac{\mu_0 n i}{2} \int_{\theta_1}^{\theta_2} \sin\theta \cdot d\theta$$

$$= \frac{\mu_0 n i}{2} (\cos\theta)_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{\mu_0 n i}{2} (\cos\theta_1 - \cos\theta_2)}$$

Different Cases:-



(i) For an infinitely long and thin solenoid,

$\theta_1 = 0$, $\theta_2 = \pi$ (90)

$$\therefore B = \frac{\mu_0 n i}{2} (\cos 0 - \cos\pi)$$

$$B = \frac{\mu_0 n i}{2} (1+1) = \frac{\mu_0 n i}{\cancel{2}} \cdot \cancel{2} \Rightarrow \boxed{B = \mu_0 n i}$$

* Mag. field inside a thin long solenoid is constant throughout the central region.

(ii) if the point is at an end of the solenoid, then $\theta_1 = 0$; $\theta_2 = \pi/2$

$$\therefore B = \frac{\mu_0 n i}{2}\left[\cos 0 - \cos \pi/2\right] = \frac{\mu_0 n i}{2}\left[\cos 0 - \cos \pi/2\right]$$

$$= \frac{\mu_0 n i}{2}\left[1-0\right] = \frac{\mu_0 n i}{2}$$

$$\therefore \boxed{B = \frac{\mu_0 n i}{2}}$$

Mag. field at the end is half of that in the middle of the solenoid.

The variation of the field with distance from the centre is shown fig.

Note :-

if $N \rightarrow$ Total no. of turns

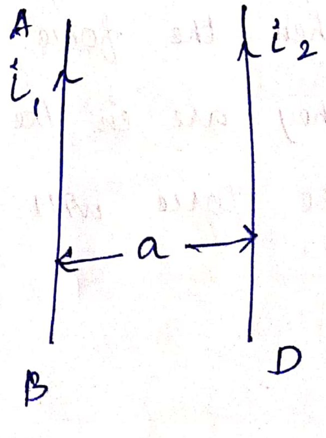
$l \rightarrow$ length of the solenoid.

The field intensity,

$$B = \frac{\mu_0 n i}{2l}\left[\cos\theta_1 - \cos\theta_2\right]$$

Force between two parallel conductors :-

* Let us consider two long straight conductors AB and CD carrying currents i_1 and i_2.
* $a \rightarrow$ parallel distance between the conductors.
* Due to the current in the conductor AB, a mag. field is produced around it.



$$B = \frac{\mu_0 i_1}{2\pi a} \ . \ wb/m^2 \quad \text{—(1)}$$

The current carrying conductor CD is in the mag. field. Hence it will experience a force.

The force experienced per unit length of the conductor CD is,

$$\boxed{F = B i_2 \ N/m} \longrightarrow ②$$

Sub. the value of B,

we get, Force $\boxed{F = \frac{\mu_0 i_1 i_2}{2\pi a} \ N/m}$ — ③

This force will act in the left hand direction. ||ly the conductor AB will experience a force which will act in the right hand direction. Thus there is a force between two current carrying conductor.

The force experienced by the conductor is,

$$\boxed{F = \frac{\mu_0 i_1 i_2}{2\pi a} \ . N/m} \longrightarrow ④$$

if the direction of the current in the 2 conductors are in the same direction, then the force will be attractive, and if they are in the opposite direction, the force will be repulsive.

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Ohm's Law :- The current flow thro' a conductor depends upon the potential difference applied.

Electric Current :-

* Is a Measure of the amount of electric charge transferred per unit time.
* It represents the flow of electron through a conductive material.

$$I = \frac{\Delta Q}{\Delta t}$$

* The magnitude of current is measured in Amperes. (Amps/A)

1 ampere = 1 coulomb/second.

Voltage :

Voltage is the electrical force, or 'pressure', that causes current to flow in a circuit.

Voltage → Volts (V)

A Voltage difference of 1 volt means 1 amp of current does 1 Joule of work in 1 Second.

Resistance :-

* Resistance measures how difficult it is for current to flow.
* The total amount of electrical resistance in a circuit determines the amount of current that in the circuit for a given voltage.
* The more resistance the circuit has, the less current that flows.

Units : Ohm (Ω).

Resistor → used to control the amount of current flowing in a circuit.

Ohm's law :- Statement :-

The current through a conductor is proportional to the potential difference between its ends, provided the temperature of the conductor remains constant.

potential difference $\propto$ current

$V \propto I$ (or) $\frac{V}{I} = R$ (constant)

$$\boxed{V = IR}$$

The ratio of voltage to current is called Resistance.

$R \rightarrow$ Resistance of the conductor.

$\frac{\text{Voltage}}{\text{Current}} = \text{Resistance}$

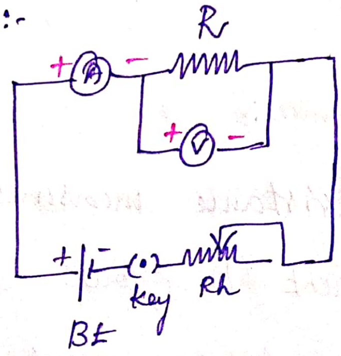
V = Volt

I = ampere

R = ohm

Verification of ohm's Law :-

Circuit Diagram :-



A - Ammeter

R - Resistance

V - Voltmeter

Bt - Battery

Rh - Rheostat

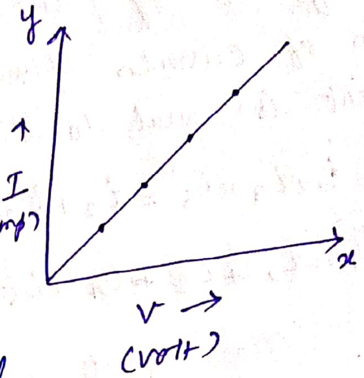
The current through the circuit can be varied with the help of rheostat connected in series with the battery.

Using the rheostat the current thro' the circuit is kept at a particular value I.

Now the voltmeter reading is noted. Let it be 'V'.

* Similarly by changing the current, for each current the p.d across the resistance is noted. For each current V/I is calculated. It is found to be constant. This verifies the Ohm's law.

* A graph drawn between voltage and current, shows a straight line pattern.



* The straight line indicates a relationship and is named as Ohm's law.

Kirchhoff's Law :-

Kirchhoff's circuit laws are two equalities that deal with the conservation of charge and energy in electrical circuits.

There basically two Kirchhoff's laws:

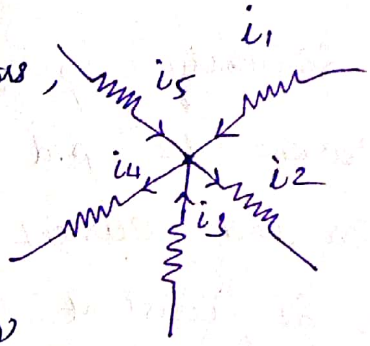
1. Kirchhoff's First law (or) Kirchhoff's Current law (KCL)

2. Kirchhoff's Second law or " Voltage law (KVL)

(i) Kirchhoff's Current law: Based on principle of conservation of electric charge.

(ii) Kirchhoff's Voltage law: Based on principle of conservation of energy.

(i) Kirchhoff's First law :-



In any network of conductors, in an electrical circuit, the algebraic sum of currents meeting at any point is zero or sum of currents flowing towards a point is equal to the sum of currents flowing away from it.

$$i_1 + i_3 + i_5 = i_2 + i_4$$

(or)

$$i_1 - i_2 + i_3 + i_4 + i_5 = 0 \quad \text{or} \quad \Sigma I = 0$$

The current entering any junction is equal to the current leaving that.

The 1st law is based on the principle that in an electric circuit, at any point the charge cannot be accumulated.

(ii) Kirchhoff's Second law :-

Kirchhoff's lood rule.

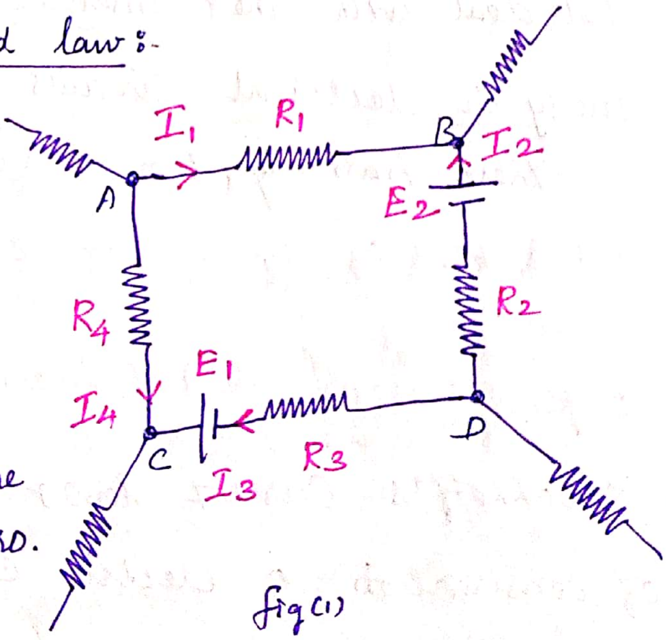


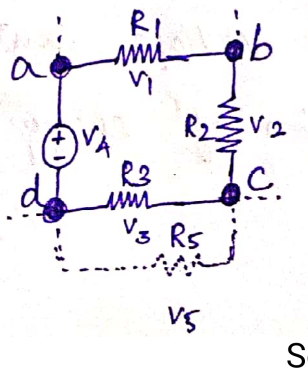
fig (1)

$$V_1 + V_2 + V_3 - V_4 = 0$$

The sum of all the voltages around the loop is equal to zero.

$$\Sigma V_{emf} = I \Sigma R$$

fig (2).



* In a closed path of networks of conductors, the algebraic sum of the products of resistance and current of each part of the closed path is equal to the algebraic sum of e.m.fs in the circuit.

* Consider a closed loop of circuit ABCDA as shown in fig (1).

* In a circuit the current which flows in the clockwise direction is taken as positive and the current which flows in the anticlock direction is taken as negative

* e.m.fs which sends current in the clockwise direction are takes as positive

* Send current in the anticlock direction as taken as negative

$$I_1R_1 - I_2R_2 + I_3R_3 - I_4R_4 = E_1 - E_2$$

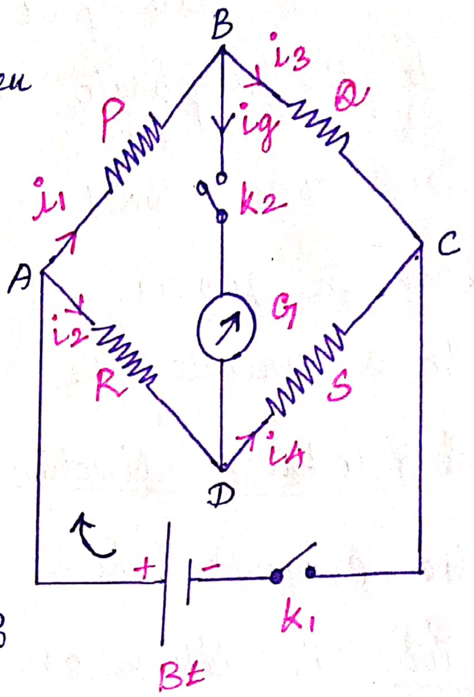
Wheatstone Bridge :-

* P, Q, R, S → four resistances.

* G → connected between the points B & D.

* When the keys are closed, a current flows in the circuit.

* Current from the cells is divided into two parts at A.



* At the junction B, (apply 1st law).

$$i_1 - i_g - i_3 = 0 \quad — (1)$$

* At the junction D,

$$i_2 - i_g - i_4 = 0 \quad — (2)$$

Bridge is Balanced, if there is no flow of current through galvanometer. For this the resistance are adjusted such that there is no deflection in G.

$\therefore i_g = 0$

From equ (1), equ (2)

$$i_1 - i_3 = 0 ; \; i_1 = i_3 \quad — (3)$$

$$i_2 - i_4 = 0 \text{ (or) } i_2 = i_4 \quad — (4)$$

⇒ Applying Second law, to the closed path ABDA,

$i_1 P + i_g G - i_2 R = 0$, — ⑤

⇒ In the closed path BCDB,

$i_3 Q - i_4 S - i_g G = 0$ — ⑥

When $i_g = 0$, eqn. ⑤ and ⑥ reduces to

$i_1 P = i_2 R$ — ⑦

$i_3 Q = i_4 S$ — ⑧

Dividing the eqn ⑦ & ⑧,

$\frac{i_1 P}{i_3 Q} = \frac{i_2 R}{i_4 S}$ But $i_1 = i_3$ and $i_2 = i_4$

$\therefore \boxed{\frac{P}{Q} = \frac{R}{S}}$ This is the <u>condition of a balanced Wheatstone Bridge</u>.

* If the value of P, Q and R are known, the value of S can be calculated.

* Used for the <u>determination of unknown resistances.</u>

* Metre Bridge, Post office Box and Carey Foster's Bridge work on this principle.

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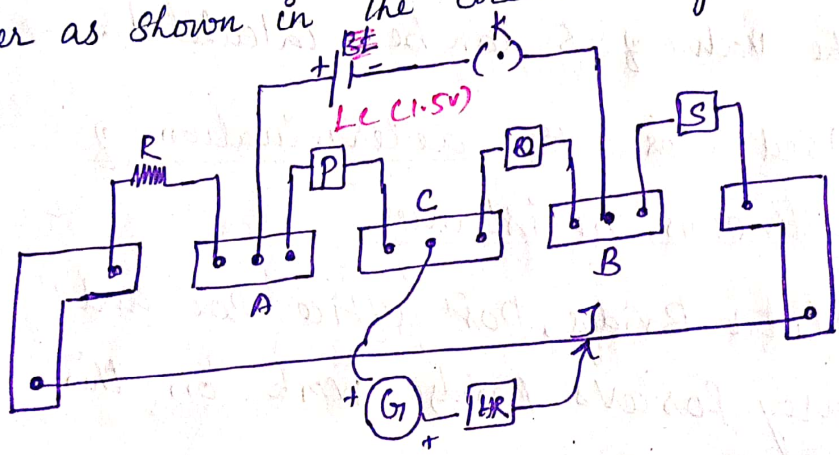
Carey Foster's Bridge : Electrical circuit.

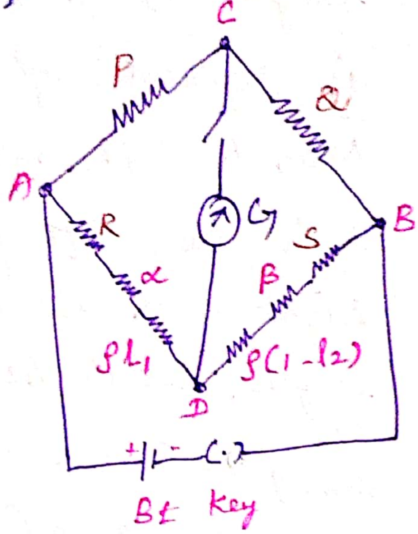
* Improved form of Metre bridge.
* More Sensitive
* To determine between two nearly equal resistances.
* If the value of one resistance is known, other can be calculated.
* Used to measure very small resistances.
* It works on the same principle as Wheatstone's bridge which consists of four resistances P, Q, R and S that are connected to each other as shown in the circuit diagram.



* It consists of a straight uniform wire of managing exactly 1 metre long (AB). The wire is stretched on a wooden board. The ends A and B are joined to thick copper strips of low resistance.

* Let α and β be the end resistances at the ends A and B respectively. Let 'ρ' be the resistance per unit length. The Resistance R is in left gap and

S in the right gap. Now l_1 be the balancing length. In this condition, the equivalence wheatstone bridge is shown in fig. When the bridge is balanced.



$$\frac{P}{Q} = \frac{R+\alpha+l_1\rho}{S+\beta+(1-l_1)\rho} \longrightarrow (1)$$

Now R & S are interchanged, & the balancing length l_2,

$$\frac{P}{Q} = \frac{S+\alpha+l_2\rho}{R+\beta+(1-l_2)\rho} \longrightarrow (2)$$

Comparing equ's (1) & (2),

we get,

$$\frac{R+\alpha+l_1\rho}{S+\beta+(1-l_1)\rho} = \frac{S+\alpha+l_2\rho}{R+\beta+(1-l_2)\rho}$$

Add 1 on both sides, we get,

$$\frac{R+\alpha+l_1\rho+S+\beta+(1-l_1)\rho}{S+\beta+(1-l_1)\rho} = \frac{S+\alpha+l_2\rho+R+\beta+(1-l_2)\rho}{R+\beta+(1-l_2)\rho}$$

$$\frac{R+S+\alpha+\beta+\rho}{S+\beta+(1-l_1)\rho} = \frac{R+S+\alpha+\beta+\rho}{R+\beta+(1-l_2)\rho}$$

$$\therefore S+\beta+(1-l_1)\rho = R+\beta+(1-l_2)\rho$$

$$S-l_1\rho = R-l_2\rho$$

$$R-S = \rho(l_2-l_1)$$

$$\boxed{R = S+\rho(l_2-l_1)}$$

Knowing ρ, l_1, l_2 and $(R-S)$ can be calculated.
If S is known R can be calculated.

Determination of ρ :-

To determine the resistance per unit length of the Bridge wire, the resistance 'R' is replaced by a copper strip ($R=0$). Now, to find balancing length l_2. Now, keeping 'S in the left gap and copper strip in the right gap. We have to find the balancing length. Let it be 'l_2'.

The value of $P = Q$

$$R = S + \rho(l_2 - l_1)$$

Here $R=0$

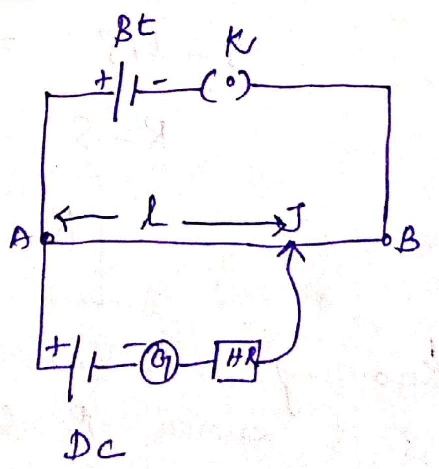
$$\therefore \boxed{\rho = \frac{S}{(l_1 - l_2)}}$$

The experiment is repeated for different values of S. The mean value of ρ is calculated.

Potentiometer :-

* Potentiometer is a device.
* Used to determine potential difference accurately.
* It is also used to measure current.

(i) primary circuit :



Construction :

It consists of 10 segment of a uniform wire of magnanin or constantan, each one metre long.

The segments are stretched parallel to each other on a horizontal wooden board. The ends of the wires are connected to copper strips of zero resistance. The ends are fitted with binding screws for connection. Using a movable jockey contact can be made at any point of the wire.
A metre scale is fixed on the wooden board parallel to the segment of the wire.

Principle of potentiometer:-

* A steady current is maintained across the wire AB by a battery Bt.
* The battery, key and the potentiometer wire are connected in series forms the primary circuit.
* The +ve terminal of a primary cell of e.m.f is connected to the jockey thro' a galvanometer G and a high resistance HR. This forms the secondary circuit.
* Let contact be made at any point J on the wire by jockey.
* If the potential difference across AJ is equal to the e.m.f of the cell (DC) then no current will flow through the galvanometer and it will show zero deflection.

⇒ AJ is the balancing length l.

⇒ The p.d across AJ is equal to Irl where I is the current flowing through the wire and 'r' is the resistance per unit length of the wire.

Hence $E = i \rho l$

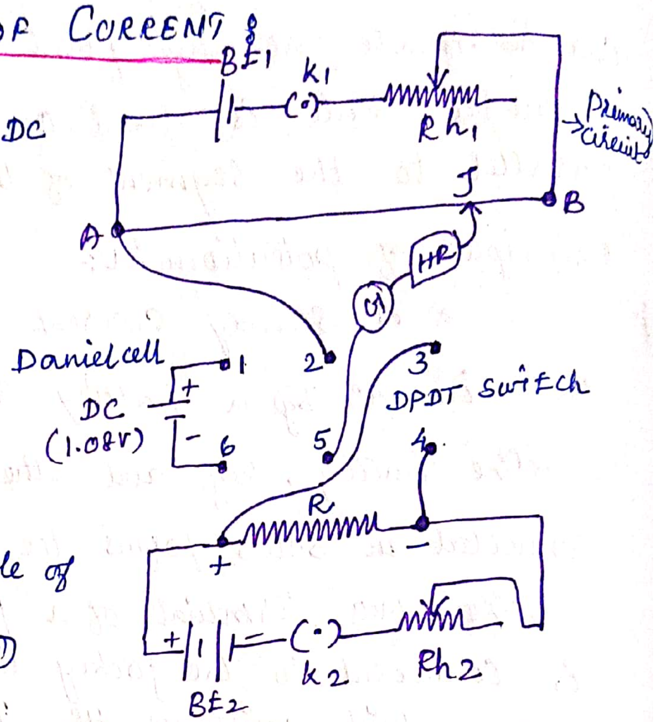
Since I and r are constants, $E \alpha l$.

$E \propto l$

The emf of the cell is directly proportional to the balancing length

This is the principle of the potentiometer.

(a) MEASUREMENT OF CURRENT :



* First including the DC in the potentiometer, the balancing length is determined.
* Let the Balancing length be l_0.
* According to the principle of potentiometer,

$$\boxed{1.08 \propto l_0} \quad —(1)$$

* Next the p.d across the Standard resistance R is included in the main circuit and the balancing length l is determined.
* If 'i' is in the current flow through the standard resistance, the p.d across R is iR.

$$\boxed{iR \propto l} \quad —(2)$$

Dividing equ. (2) by (1),

$$\frac{iR}{1.08} = \frac{l}{l_0} \quad —(3)$$

$$\boxed{i = \frac{1.08}{R} = \frac{l}{l_0}} \quad —(4)$$

Using equ (4), the current in the circuit can be calculated.