UNIT-I:

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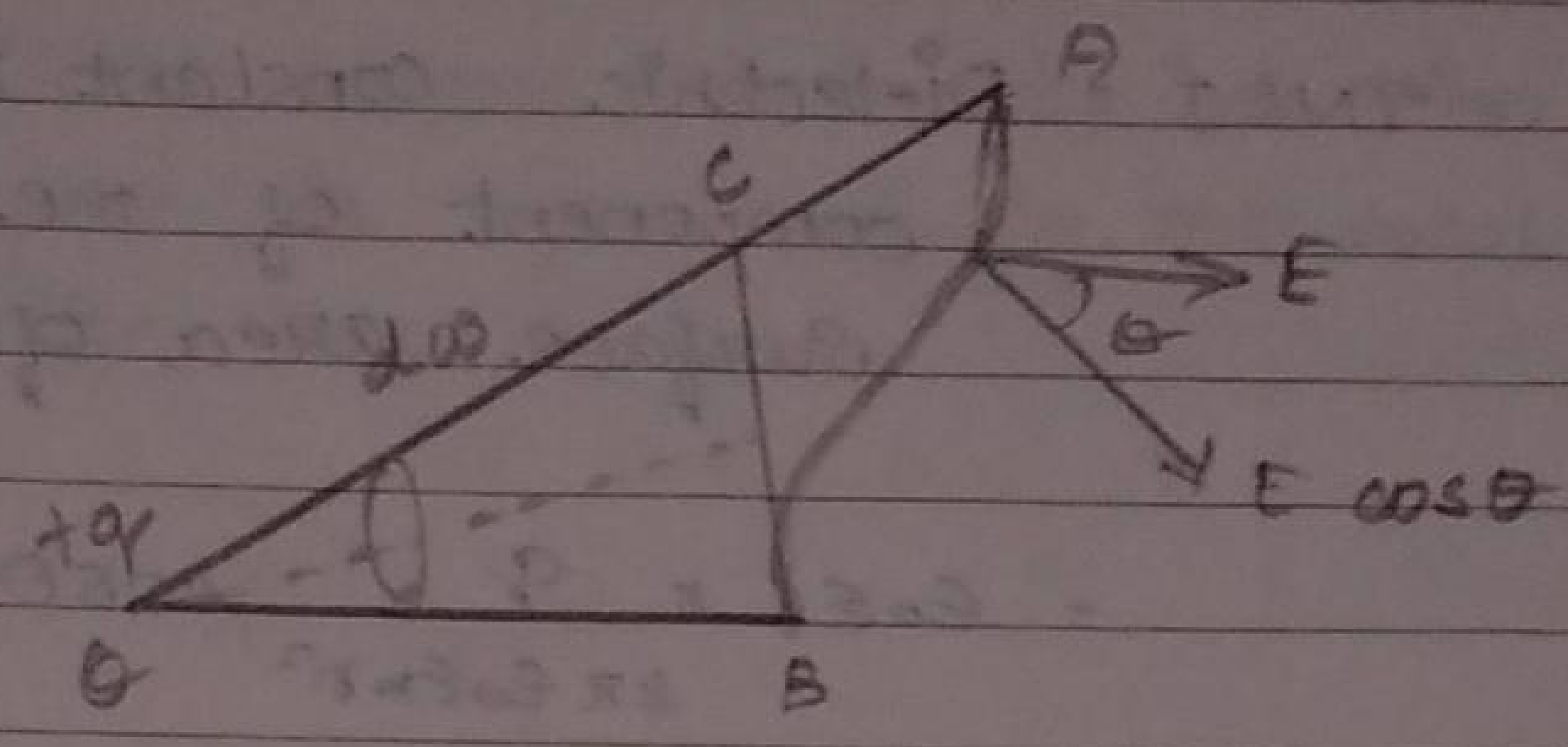
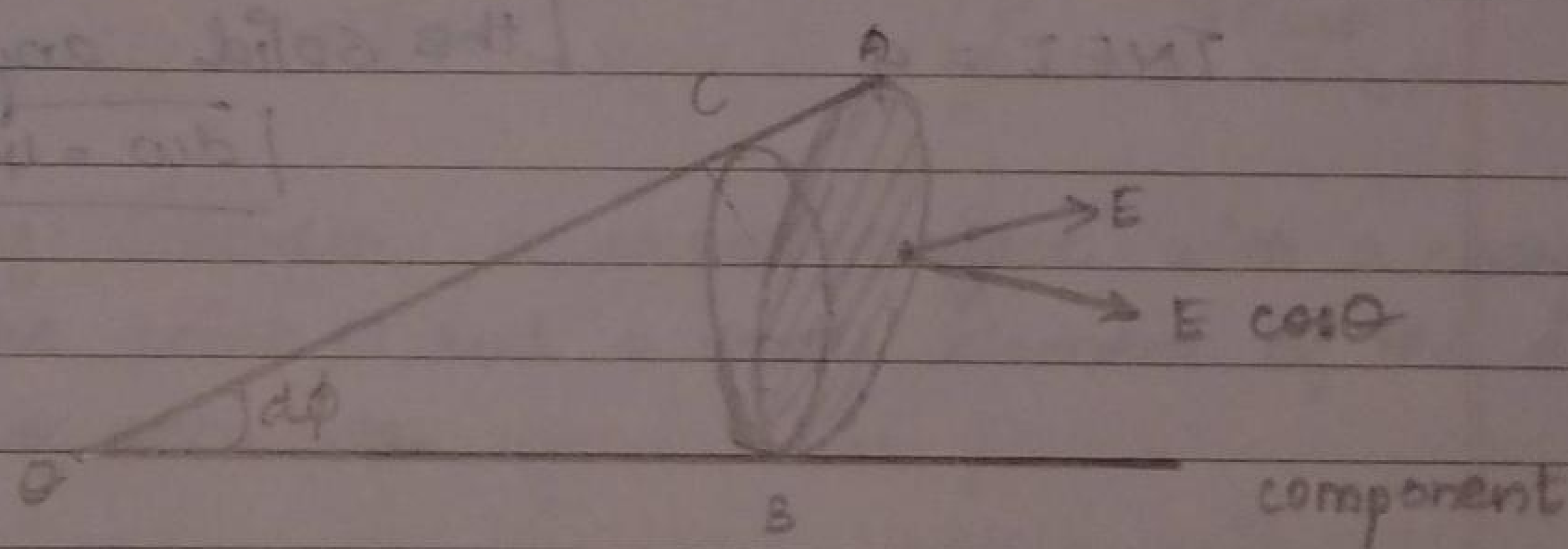
ELECTROSTATICS

Gauss's Theorem.



* Total normal electrical induction over a closed surface is equal to Σq (i.e) total charge present inside the surface.

* Consider a closed surface with a charge q at the point "O", small elemental area of surface AB of area ds.



Electric Intensity at a point on the surface AB $\Big\} = E = \dfrac{q}{4\pi \varepsilon_0 \varepsilon_r r^2}$

Component of the Intensity $\perp^r$ to the surface.

$$= E\cos\theta = \frac{q}{4\pi\epsilon_0\epsilon_r r^2}\cos\theta$$

TNEI = Dielectric constant × component of the intensity $\perp^r$ to surface × area of the surface.

$$= \epsilon_0\epsilon_r \times \frac{q}{4\pi\epsilon_0\epsilon_r r^2}\cos\theta \times AB$$

$$= \left(\frac{q}{4\pi}\right)\frac{AB\cos\theta}{r^2} = \frac{q\,d\omega}{4\pi}$$

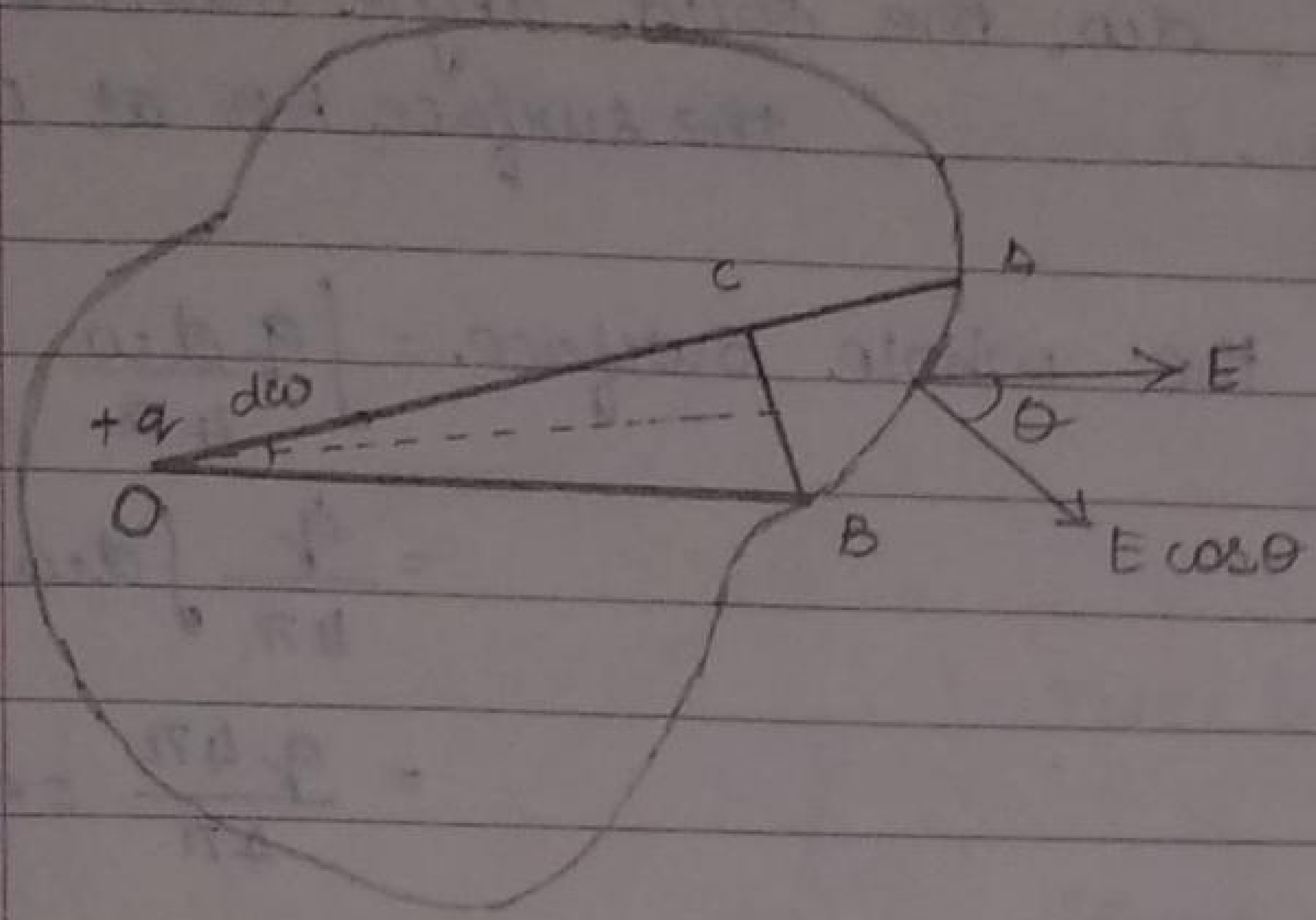
$$\left[\frac{AB\cos\theta}{r^2} = d\omega \text{ solid angle}\right]$$

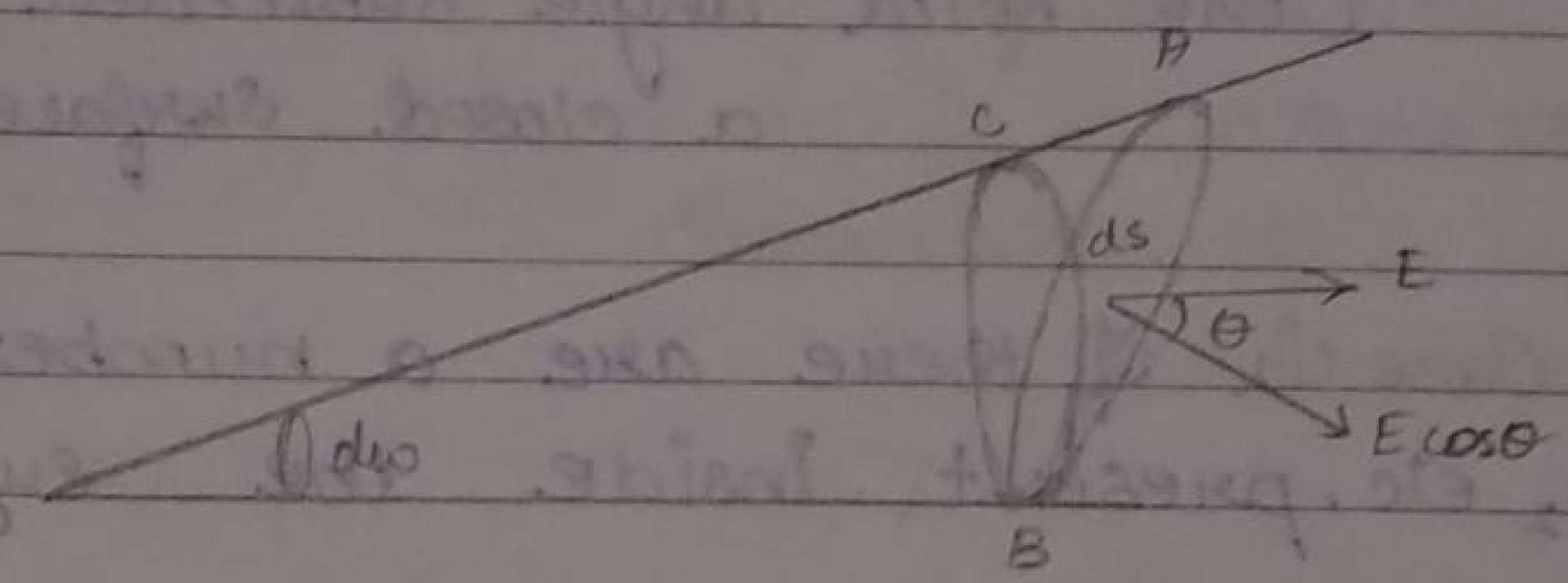
$$\text{TNEI} = \int\frac{q\,d\omega}{4\pi} = \frac{q}{4\pi}\int d\omega = \frac{q}{4\pi}(4\pi)$$

$$\text{TNEI} = q \qquad \left[\text{the solid angle } \boxed{d\omega = 4\pi}\right]$$

Gauss's Theorem.

⇒ Total normal electrical induction over a closed surface is equal to Σq the total charge present inside the surface.





Proof: Consider a closed surface with a charge q at the point O and a small element of the surface AB of area ds.

Electric intensity at a point on the surface AB $= E = \dfrac{q}{4\pi\varepsilon_0\varepsilon_r r^2}$

Component of the intensity perpendicular to the surface

$$= E\cos\theta = \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2}\cos\theta$$

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TNEI over this elementary surface

= Dielectric constant × component of the intensity perpendicular to the surface × area of the surface

$$= \varepsilon_0 \varepsilon_r \times \frac{q}{4\pi\varepsilon_0\varepsilon_r r^2} \cos\theta \times AB = \left(\frac{q}{4\pi}\right)\frac{AB\cos\theta}{r^2} = \frac{q\,d\omega}{4\pi}$$

[$\frac{AB\cos\theta}{r^2} = d\omega$, the solid angle subtended by the surface AB at O]

TNEI over the whole surface $= \int \frac{q\,d\omega}{4\pi}$

$$= \frac{q}{4\pi}\int d\omega$$

$$= \frac{q\,4\pi}{4\pi} = q$$

(the solid angle subtended at a point inside a closed surface $= 4\pi$)

Case.(i). If there are a number of charges q_1, q_2 etc. present inside the surface, the

$$TNEI = q_1 + q_2 + q_3 + \dots = \Sigma q.$$

Case.(ii). If the charge q is outside the surface, the total normal electrical induction over the closed surface is zero. (i) At A, TNEI inwards $= -\frac{1}{4\pi} q\,d\omega$. (ii) At B, TNEI outwards $= +\frac{1}{4\pi} q\,d\omega$.

(iii) At C, TNEI inwards $= -\frac{1}{4\pi} q\,d\omega$. (iv) At D, TNEI outwards $= +\frac{1}{4\pi} q\,d\omega$

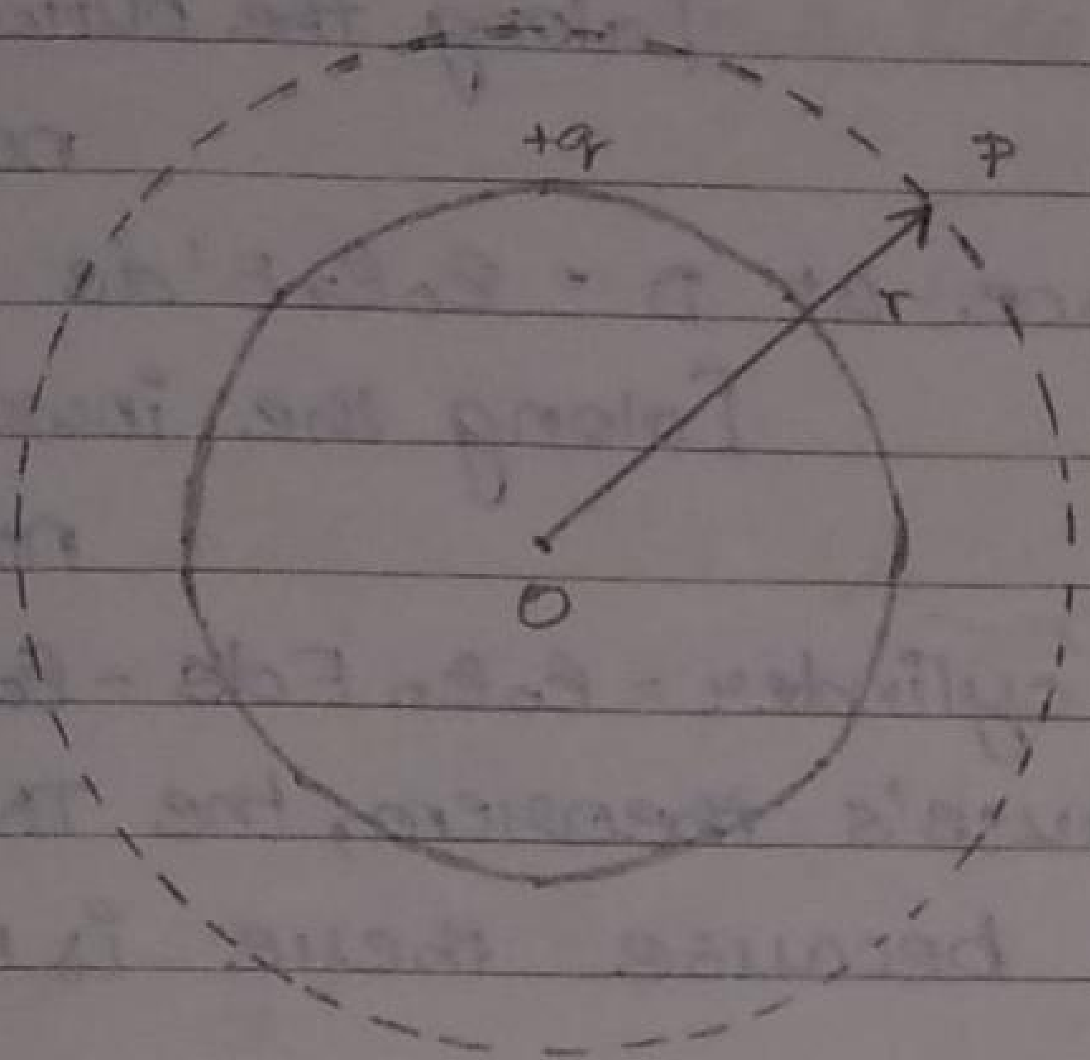
$$\therefore \frac{1}{4\pi}\Sigma q\,d\omega = 0$$

Applications of Gauss's Theorem

(1) Electric intensity at a point due to a charged sphere. Consider a point P at a distance r from the centre of a sphere.

Let the electric intensity at P be E.

Charge on the sphere $= q$. TNEI according to Gauss's theorem.



$= q$ ... (i)

TNEI is also $= \epsilon_0 \epsilon_r EA$

$= \epsilon_0 \epsilon_r \times E \times 4\pi r^2$ ... (ii)

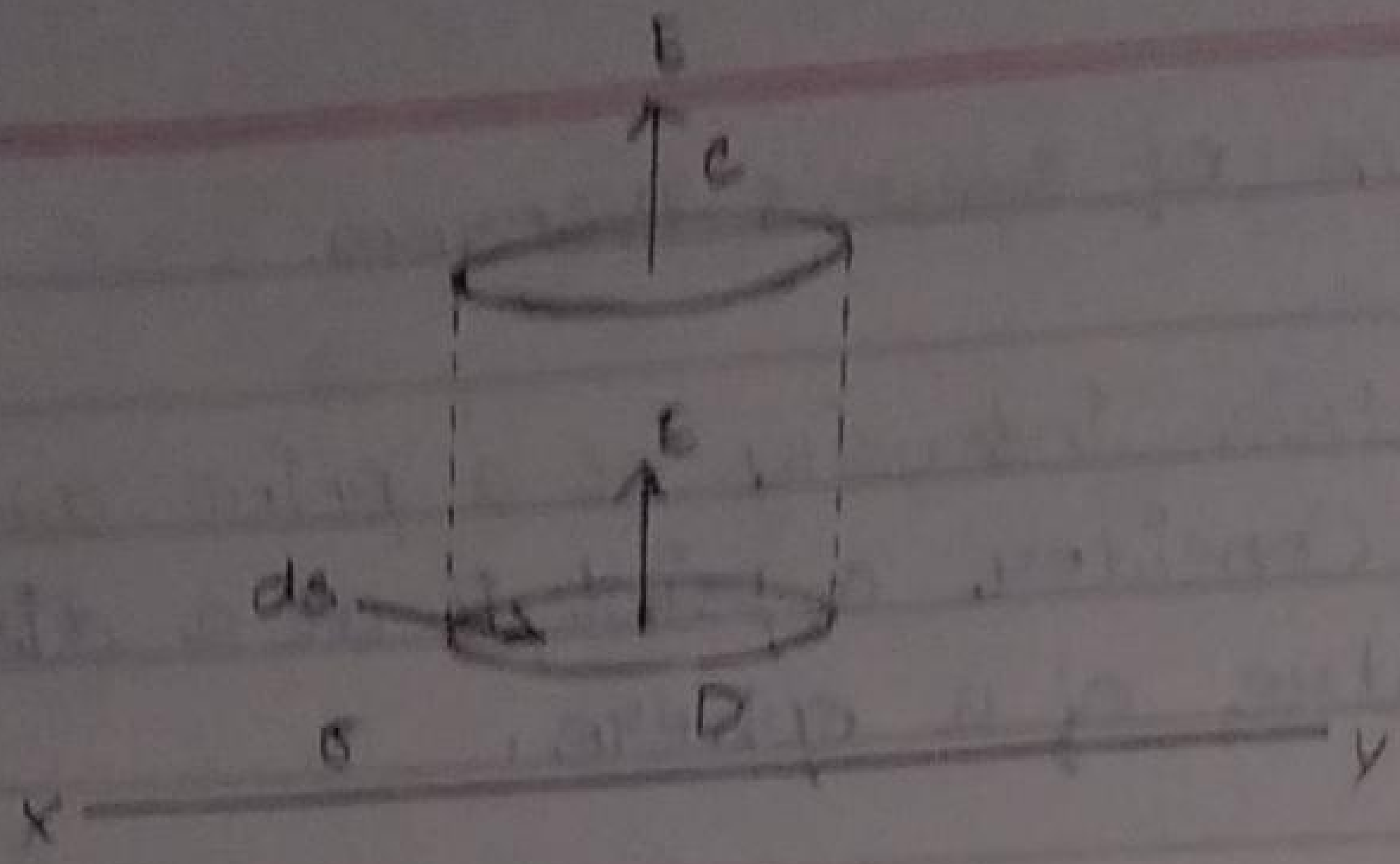
(Surface area of the sphere of radius r $= A = 4\pi r^2$)

From (i) and (ii)

$$q = \epsilon_0 \epsilon_r E \cdot 4\pi r^2$$

$$E = \frac{q}{4\pi\epsilon_0\epsilon_r r^2} \quad \ldots (iii)$$

(2) Electric Intensity at a point near an infinite plane charged conductor. Let XY be a plane charged conductor having surface density of charge σ. C and D are two points very near the conductor. Let E and E' be the electric intensities at C and D respectively.



Consider an imaginary cylinder of area of cross section = ds. TNEI over the surface at C = $\varepsilon_0 \varepsilon_r \times E \times ds$

[along the outward drawn normal]

TNEI over the surface at D = $\varepsilon_0 \varepsilon_r E' ds$

[along the inward drawn normal]

TNEI for the whole cylinder = $\varepsilon_0 \varepsilon_r E ds - \varepsilon_0 \varepsilon_r E' ds$

But according to Gauss's theorem, the TNEI over the cylinder is zero because there is no charge inside the cylinder.

$$\therefore \quad \varepsilon_0 \varepsilon_r E ds - \varepsilon_0 \varepsilon_r E' ds = 0$$

or
$$\varepsilon_0 \varepsilon_r E ds = \varepsilon_0 \varepsilon_r E' ds$$
$$E = E'$$

Hence the electric intensity at all points near the charged plane is the same.

Now consider the two points C and D on the two sides of the charged plane.

Consider an imaginary cylinder of cross-sectional area ds.

TNEI over the surface at C = $\varepsilon_0 \varepsilon_r E ds$

[along the outward drawn normal]

TNEI over the surface at $D = \varepsilon_0 \varepsilon_r E\,ds$

[along the outward drawn normal]

TNEI for the whole cylinder $= \varepsilon_0 \varepsilon_r E\,ds + \varepsilon_0 \varepsilon_r E\,ds$

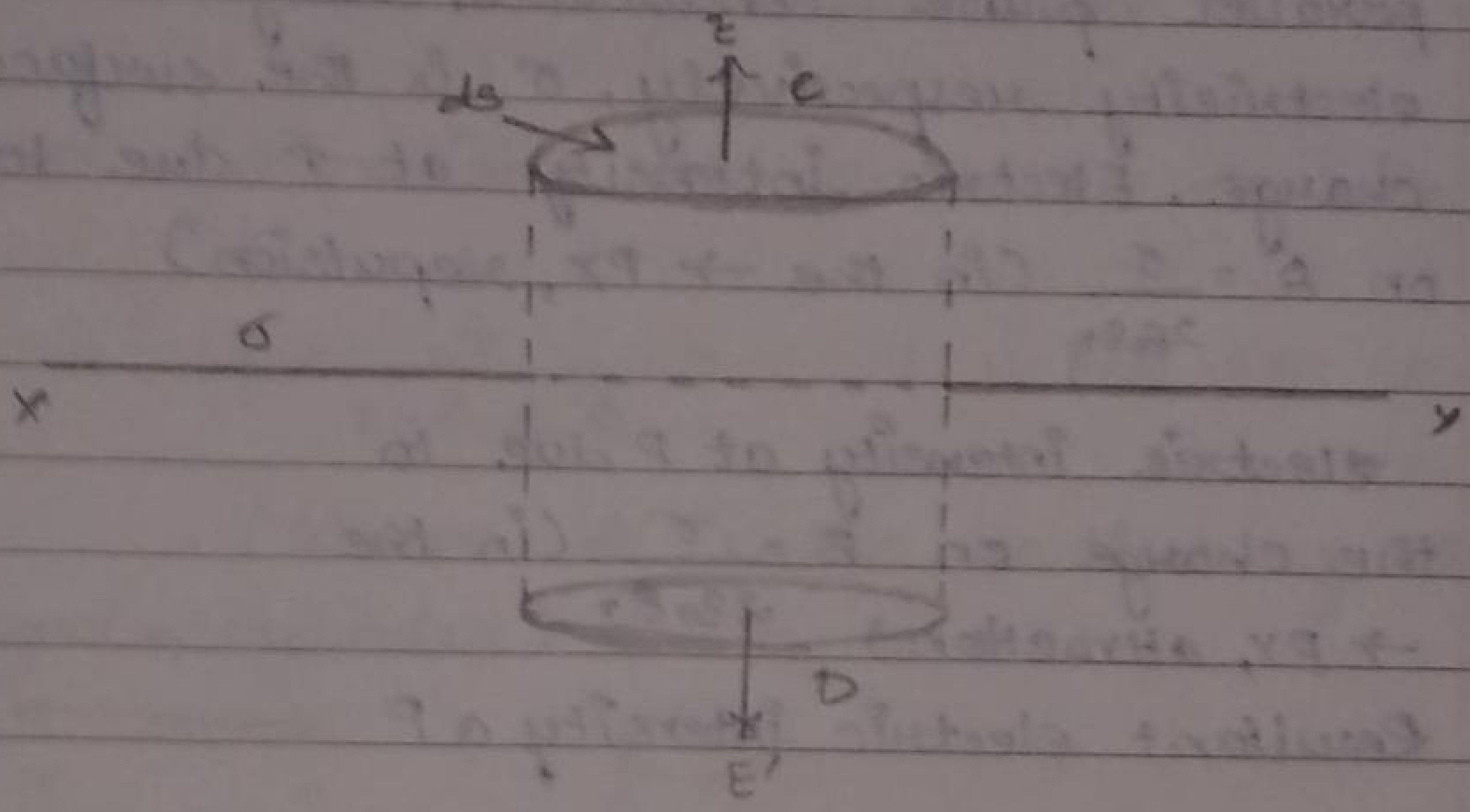
$= 2\varepsilon_0 \varepsilon_r E\,ds$

[along the outward drawn normal]

But according to Gauss's theorem, TNEI $= \sigma\,ds$ ($\sigma\,ds$ is the charge inside the cylinder)

$$2\varepsilon_0 \varepsilon_r E\,ds = \sigma\,ds \quad \ldots(i)$$

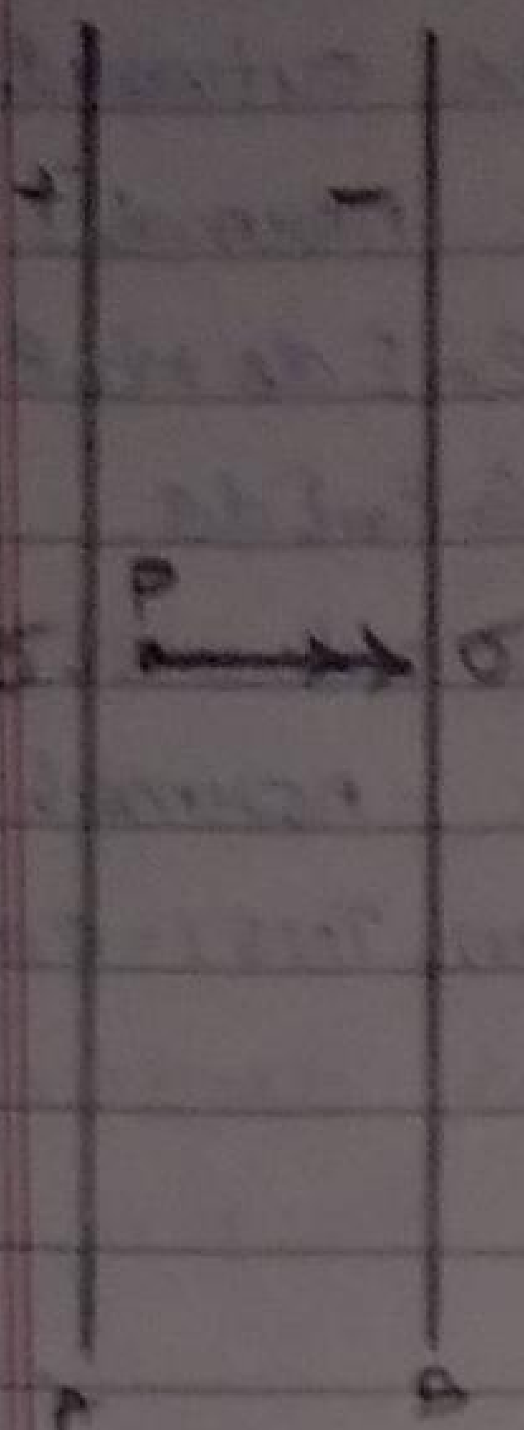
$$E = \frac{\sigma}{2\varepsilon_r \varepsilon_0} \quad \ldots(ii)$$



(3) Electric intensity at a point between two parallel plane charged conductors. A and B are two charged plane conductors. A is positively charged and B is negatively charged. Surface density of charge is σ and the dielectric constant is $\varepsilon_0 \varepsilon_r$.

Electric intensity at P due to the charge on A $= \dfrac{\sigma}{2\varepsilon_0 \varepsilon_r}$ (repulsion)

Electric intensity at P due to the charge on B $= \frac{\sigma}{2\varepsilon_r\varepsilon_0}$ (attraction)



Resultant electric intensity at P

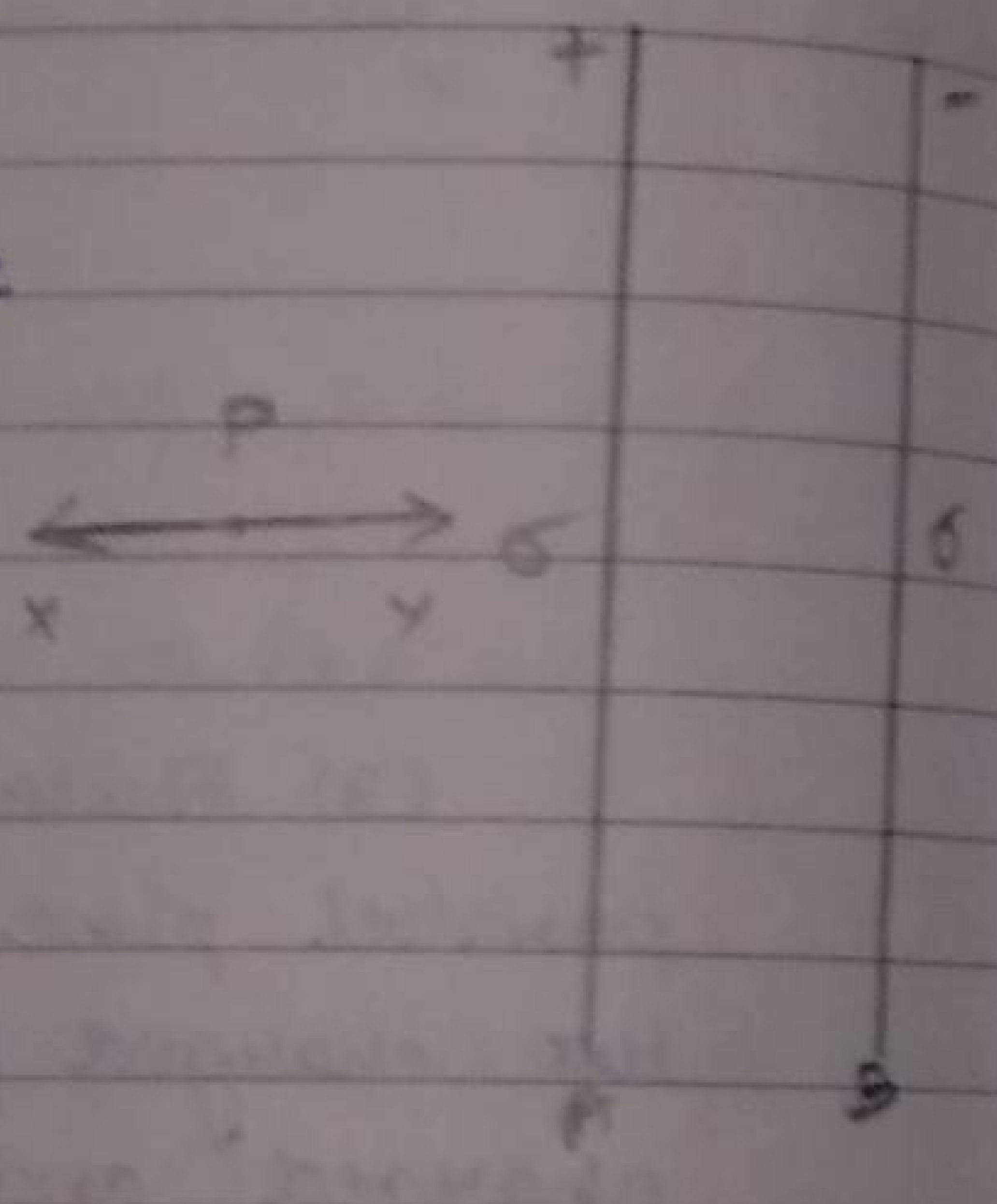
$$= \frac{\sigma}{2\varepsilon_0\varepsilon_r} + \frac{\sigma}{2\varepsilon_0\varepsilon_r} \quad \text{.....(i)}$$

$$= \frac{\sigma}{\varepsilon_0\varepsilon_r} \quad \text{.....(ii)}$$

(4) Electric intensity at a point outside two parallel plane charged conductors. Let A and B be two parallel plane conductors, charged with +ve and -ve electricity respectively, σ is the surface density of charge. Electric intensity at P due to the charge on A $= \frac{\sigma}{2\varepsilon_0\varepsilon_r}$ (in the → PX, repulsion)

Electric intensity at P due to the charge on B $= \frac{\sigma}{2\varepsilon_0\varepsilon_r}$ (in the → PY, attraction)

Resultant electric intensity a P



$$= \frac{\sigma}{2\varepsilon_0\varepsilon_r} - \frac{\sigma}{2\varepsilon_0\varepsilon_r} = 0$$

(5) Electric intensity at a point due to a uniform charged cylinder. Let AB be the charged cylinder having q units per metre. P is a point at a distance of r from the axis of the cylinder the R is the radius of the cylinder. Construct an imaginary cylinder of radius r and length l round the cylinder AB. Let E be the electric

intensity at P. Then

TNEI over the curved surface of the cylinder of radius $r = \varepsilon_0 \varepsilon_r E . 2\pi r . l$

But TNEI for the curved surface area of this cylinder according to Gauss's theorem $= ql$

[ql is the charge inside the imaginary cylinder]

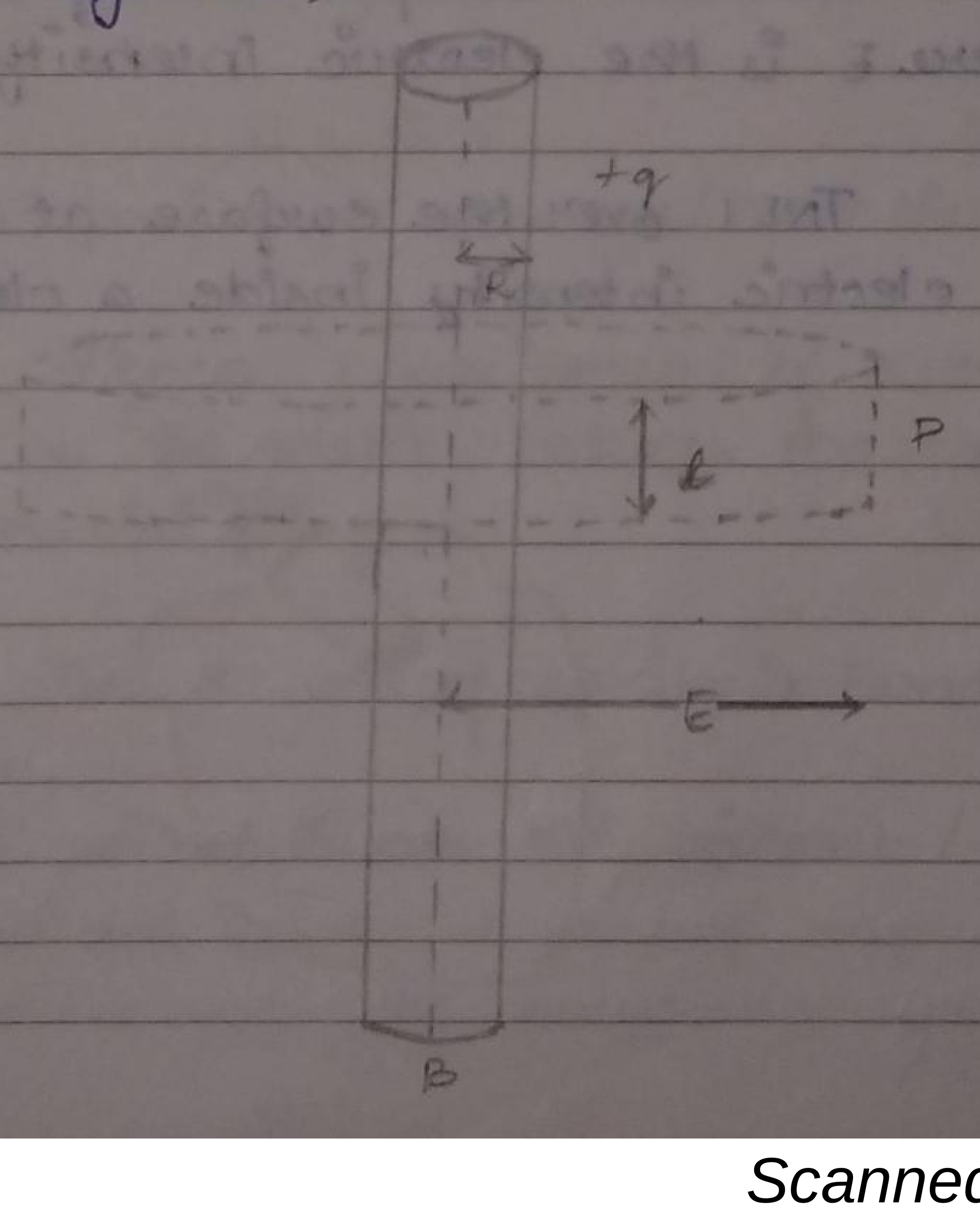
(There are no electric lines of force perpendicular to the plane surface are of the imaginary cylinder).

From (i) to (iii), $ql = \varepsilon_0 \varepsilon_r . E . 2\pi r . l$

In rationalised MKS units or SI units

$$E = \frac{q}{2\pi \varepsilon_r \varepsilon_0 R} \quad \ldots (ii)$$

(Where R is the radius of the charged cylinder)



Coulomb's law:

It states that the electric intensity at any point near a charged surface of any shape is equal to $\frac{\sigma}{\varepsilon_0 \varepsilon_r}$ where σ is the surface

of any shape is equal to $\frac{\sigma}{\varepsilon_0 \varepsilon_r}$ where σ is the

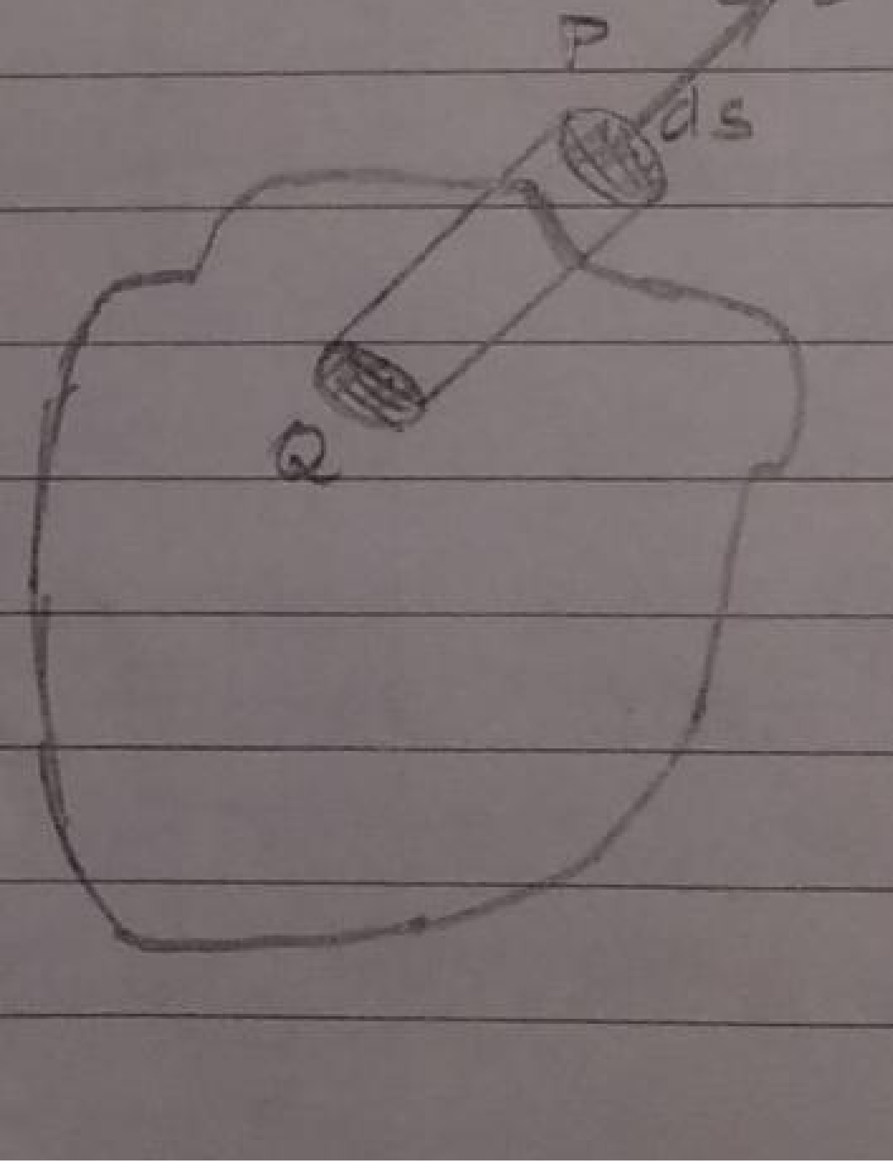
surface density of charge and $\varepsilon_0 \varepsilon_r$, is the dielectric constant of the medium.

Proof: Let ABC be the charged surface and σ the surface density of charged.

Consider two points P and Q, one outside and the other inside the surface and close to the surface. Construct an imaginary cylinder, as shown in the diagram, of cross sectional area ds.

TNEI over the plane surface at P = $\varepsilon_0 \varepsilon_r E.ds$ where E is the electric intensity at P.

TNEI over the surface at Q = 0, because the electric intensity inside a closed charged.



Capacitors

Introduction:

When charge is given to a conductor, its potential increases. If the charge is continuously increased, its potential also increases. The potential acquired by the conductor is directly proportional to charge given. At any instant of time if Q is the charge and V the potential then

$$Q \propto V \qquad \ldots (1)$$

or $$Q = CV \qquad \ldots (2)$$

where C is a constant whose value depends upon nature of the conductor and its surrounding medium. This constant is called the capacitance of the conductor.

$$\therefore C = Q/V \qquad \ldots (3)$$

Capacitance = Charge/Potential

Hence capacitance of a conductor is defined as the ratio between the charge and the potential.

The S.I. unit of capacity is Farad. From definition,

1 Farad = 1 coulomb / 1 Volt

The partical unit of capacity of farad is too larger. Hence for practice much smaller units are used.

1 milli farad (mf) = 10^{-3} F

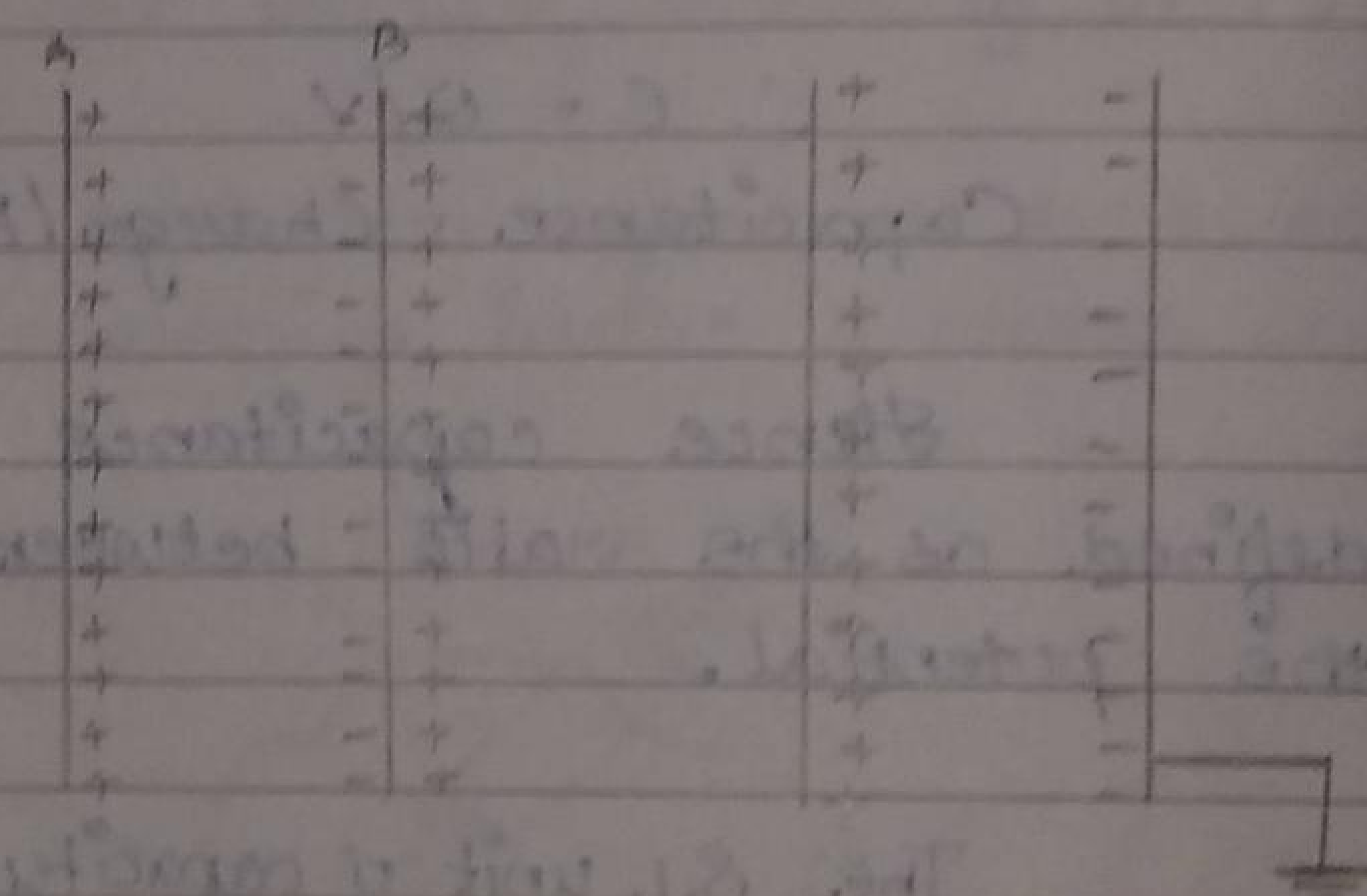
1 micro farad (μF) = 10^{-6} F

1 pico farad (pF) = 10^{-12} F

Principle of a capacitor:

A positive charge of +Q coulomb is given to conductor A. If its potential is V, the Capacitance of A is $C = Q/V$ Farad.

Now a conductor B is placed near the conductor A as shown in diagram. The space between the two conductors is filled with dielectric medium. Due to Induction, the nearer face of plate B becomes negatively charged while the farther side face becomes positively charged.



Induced negative charge decreases the potential of A while the induced positive charge increases the potential. As the negative

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charge is nearer, its effect is greater than the positive charge. Hence the resultant potential of A decreases. To bring A to its original potential, more charges should be given to A. Now if the charge on A is Q_1

$$C_1 = Q_1 / V$$

$$Q_1 > Q. \text{ Hence } C_1 > C$$

Thus the capacitance of A increases when another conductor is placed near the charged conductor.

If the plate B is earthed, the induced positive charge will disappear. Hence there is further decrease in the potential of A. In order to have the same potential, much more charges should be given. Now if the charge on A is Q_2.

$$C_2 = Q_2 / V$$

$$Q_2 > Q_1 > Q. \text{ Hence } C_2 > C_1 > C$$

Thus bringing of an earthed plate near a charged plate increases its capacitance of the plate. A device by which the capacity of a conductor is increased is called condenser. A condenser or capacitor consists of two conductors, one charged and the other is earthed.

$$\therefore C = \frac{2\pi \varepsilon_0 \varepsilon_r}{(\log_e R - \log_e r)} \quad \ldots (5)$$

$$\therefore C = \frac{2\pi \varepsilon_0 \varepsilon_r}{2.3026 \times [\log R - \log r]} \quad \ldots (6)$$

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This is the capacity per unit length of the capacitor. If l is the length of the cylinder, then

$$\text{capacitance } C = \frac{2\pi\varepsilon_0\varepsilon_r l}{2.3026[\log R - \log r]} \quad \ldots(7)$$

If the medium is air, then $\varepsilon_r = 1$.

$$\therefore C = \frac{2\pi\varepsilon_0 l}{2.3026[\log R - \log r]} \quad \ldots(8)$$

Energy of a charged capacitor

The energy of a charged conductor is the amount of work done in charging it. Let us consider a conductor of charge q. Let V be the potential. Now an additional charge dq is given to the conductor. The work done for adding a charge dq is given by $V\,dq$ joule. The work that has to be done to add a finite charge Q to the conductor is

$$W = \int_0^Q V\,dq \quad \ldots(1)$$

If C is the capacitance of the conductor

$$V = q/C \quad \ldots(2)$$

$$\therefore W = \int_0^Q \frac{q}{C}\cdot dq = \frac{1}{2}\frac{Q^2}{C} \text{ Joule} \quad \ldots(3)$$

The total work done in charging the conductor is stored as potential energy. Hence the energy of the conductor is

$$E = \frac{1}{2}\frac{Q^2}{C} \text{ Joule} \quad \ldots (4)$$

But $Q = CV$

$$\therefore E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 \text{ Joule} \quad \ldots (5)$$

Also $C = Q/V$

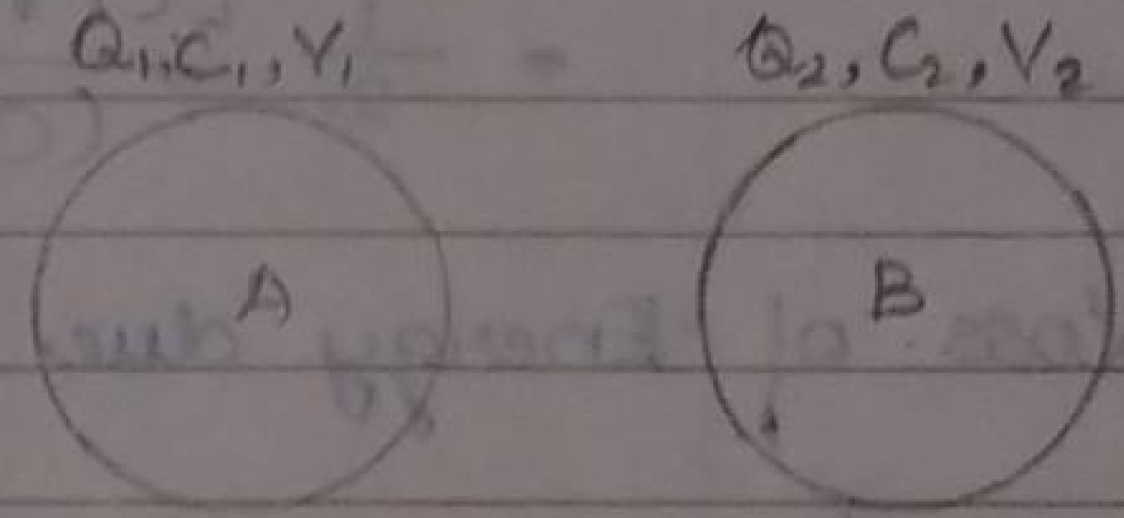
$$\therefore E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV \text{ Joule} \quad \ldots (6)$$

$\therefore$ Energy of a capacitor

$$E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV \quad \ldots (7)$$

Sharing of charges and loss of Energy.

Consider two conductors A and B of capacity C_1 and C_2, Let Q_1, Q_2 be the charges and V_1, V_2 their potentials respectively.



Energy of the capacitor A $= \frac{1}{2}C_1V_1^2$ Joule

Energy of the capacitor B $= \frac{1}{2}C_2V_2^2$ Joule

$$\therefore \text{Total Energy } E_1 = \frac{1}{2}\left[C_1V_1^2 + C_2V_2^2\right] \quad \ldots (1)$$

Now the two conductors are connected by a wire. The charge will flow from the conductor which is at higher potential to the sphere which is at lower potential. Hence both of them will attain a common potential V. Now

Total charge $Q = Q_1 + Q_2$

Total capacitance $C = C_1 + C_2$

$\therefore V = (Q_1 + Q_2)/(C_1 + C_2)$

But $Q_1 = C_1 V_1$, $Q_2 = C_2 V_2$

$$\therefore V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \quad \ldots (2)$$

After sharing of charges, the total energy

$$E_2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}(C_1 + C_2)\left[\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}\right]^2$$

$$= \frac{1}{2}\frac{[C_1 V_1 + C_2 V_2]^2}{(C_1 + C_2)} \quad \ldots (3)$$

Loss of Energy due to sharing $= E_1 - E_2$

$$= \frac{1}{2}\left[C_1 V_1^2 + C_2 V_2^2 - \frac{[C_1 V_1 + C_2 V_2]^2}{(C_1 + C_2)}\right]$$

$$= \frac{1}{2}\left[C_1 V_1^2 + C_2 V_2^2 - \frac{[C_1^2 V_1^2 + C_2^2 V_2^2 + 2 C_1 C_2 V_1 V_2]}{(C_1 + C_2)}\right]$$

$$= \frac{1}{2}\left[\frac{C_1^2 V_1^2 + C_1 C_2 V_1^2 + C_1 C_2 V_2^2 + C_2^2 V_2^2 - C_1^2 V_1^2 - C_2^2 V_2^2 - 2 C_1 C_2 V_1 V_2}{(C_1 + C_2)}\right]$$

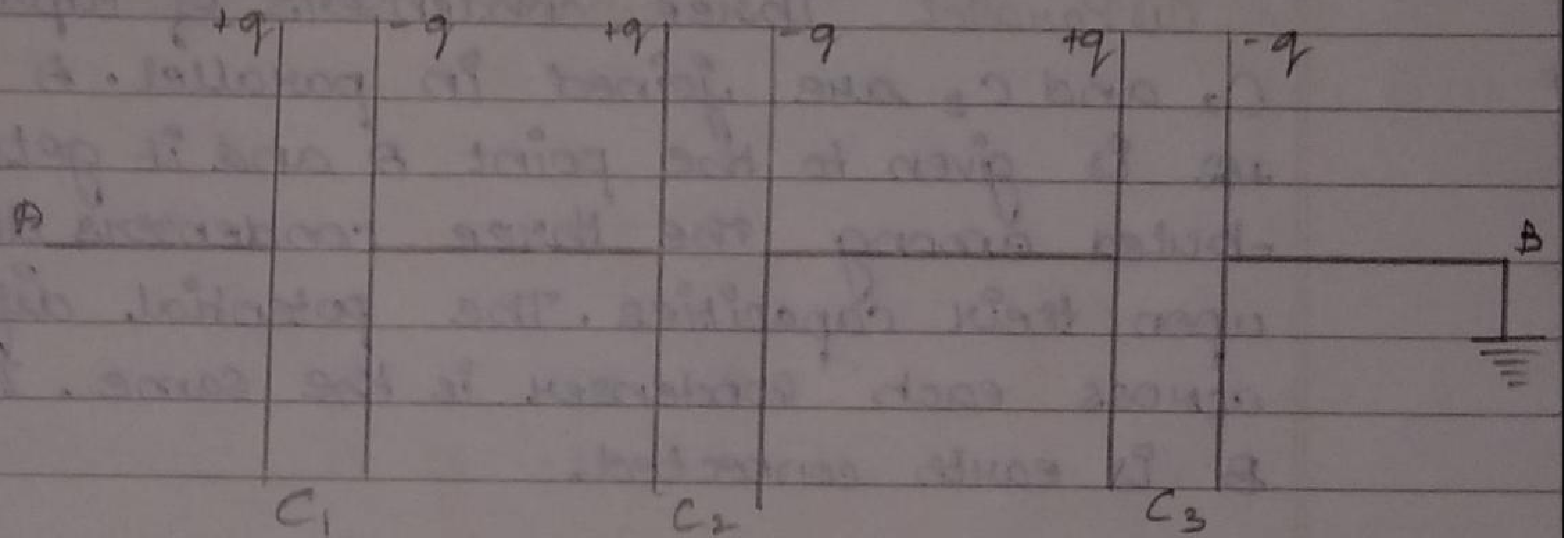
i.e. $$= \frac{1}{2}\left[\frac{C_1 C_2 (V_1^2 + V_2^2 - 2 V_1 V_2)}{(C_1 + C_2)}\right]$$

$$\therefore E_1 - E_2 = \frac{1}{2}\frac{C_1C_2(V_1-V_2)^2}{(C_1+C_2)} \text{ joule} \quad \ldots (4)$$

Whatever may be the value of V_1 and V_2, $(V_1-V_2)^2$ is always positive. $\therefore E_1 > E_2$. Hence due to sharing of charges, there will be always a loss of energy. This loss of energy appears in the form of heat.

Condensers in Series and in Parallel

(i) Series. Three condensers of capacity C_1, C_2 and C_3 are joined in series. A charge $+q$ is given to the point A and B is earth connected, P.D across $C_1 = V_1$, P.D. across $C_2 = V_2$ and P.D. across $C_3 = V_3$.



Resultant potential difference between the points A and B,

$$V = V_1 + V_2 + V_3 \quad \ldots (i)$$

Equivalent condenser will have a charge q on the plates and potential difference between the plates = V.

Hence the capacity of the equivalent condenser $= C_s = \frac{q}{V}$

$\therefore \quad V = \frac{q}{C_S}$ and $V_1 = \frac{q}{C}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$

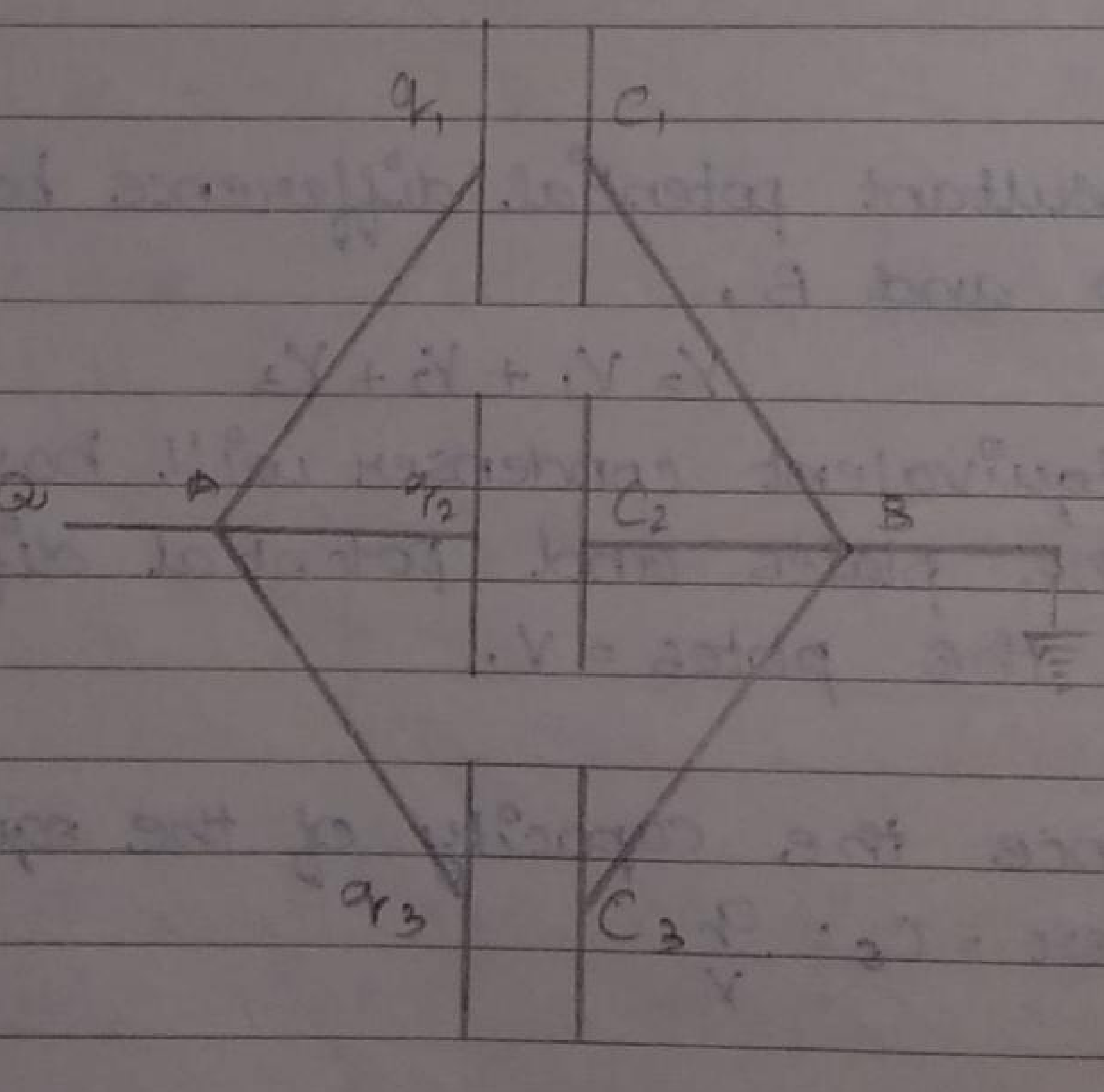
Substituting these values in (i)

$$\therefore \frac{q}{C_S} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}.$$

$$\therefore \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

When a number of condensers are connected in series, the reciprocal of the resultant capacity is equal to the sum of the reciprocals of the capacities of the individual condensers.

(ii) Parallel. Three condensers of capacity C_1, C_2 and C_3 are joined in parallel. A charge +Q is given to the point A and it gets distri-buted among the three condensers depending upon their capacities. The potential difference across each condenser is the same. The point B is earth connected.



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The potential differences between the points A and B = V.

$$Q = q_1 + q_2 + q_3$$

But $Q = C_p V$, $q_1 = C_1 V$,
$q_2 = C_2 V$, $q_3 = C_3 V$

$$C_p V = C_1 V + C_2 V + C_3 V$$
$$C_p = C_1 + C_2 + C_3$$

When a number of condensers are connected in parallel, the resultant capacity is equal to the sum of the capacities of the individual condensers.