permeability in the same manner as magnetic metals do, but unlike the ferromagnetic metals, they also have a high bulk resistivity. This means that at high frequencies, eddy currents induced within the materials are practically nonexistent, and high-Q coils can be made. Typical values for μ are around 100 and for resistivity 10,000 Ω-cm. A high length-to-diameter ratio for the rod gives a high permeability, which is desirable.

The size of coil is a compromise among several factors. If the coil is too long compared to the rod length, the change of permeability with temperature will cause a noticeable change in the inductance. If it is too short, the Q will be low. Positioning the coil on the core is critical as well, since the effective permeability is a function of position on the rod, ranging from a maximum at the center to a minimum at either end. The coil is usually placed near the quarter-point, allowing adjustment in either direction to trim the coil inductance. When more than one coil is mounted on the same rod, they must be placed at opposite ends to minimize interaction between them.

The coil of wire on the ferrite rod is basically a modified loop antenna, so the induced maximum emf appearing at its terminals is given by

$$V_s = \omega BANF\mu_r \tag{16.14.3}$$

where F = modifying factor accounting for coil length, ranging from unity for short coils to about 0.7 for one that extends the full length of the rod

μ_r = effective relative permeability of the rod, as measured for the actual coil position

A = rod cross-sectional area

An expression for the effective length of a ferrite rod antenna can be derived by combining Eqs. (B.4), (B.8), (16.9.1), and (16.14.3) to give

$$\ell_{\text{eff}} = \frac{2\pi ANF\mu_r}{\lambda} \tag{16.14.4}$$

Since the voltage appearing at the terminals is of more importance in a receiving antenna, the factor $Q\ell_{\text{eff}}$ is often given as a figure of merit for rod antennas. The directional properties of the ferrite-rod antenna are similar to those of the loop antenna, although the null may not be quite so pronounced.

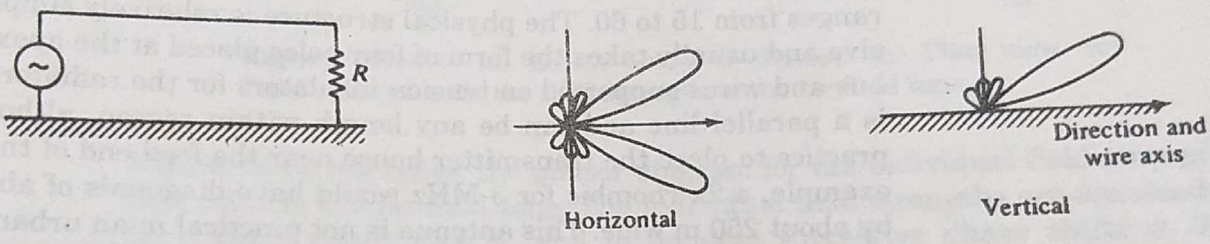
Nonresonant Antennas

Long-wire Antenna

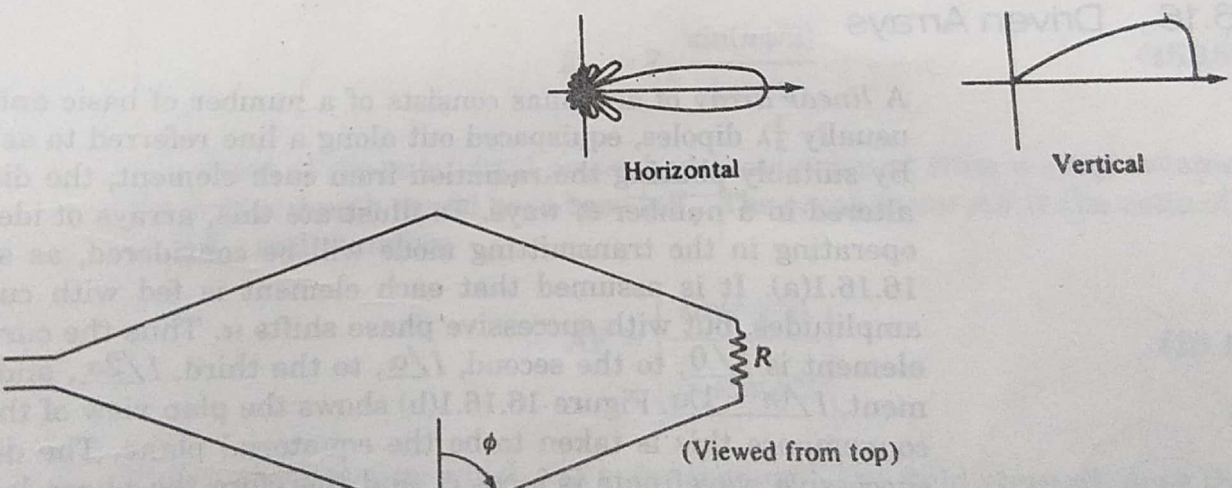
The *long-wire antenna* is just that, a wire several wavelengths in length that is suspended at some height above the earth. The wire is driven at one end and has a resistive termination at the remote end that is matched to the characteristic impedance of the line at that end. This forms a transmission line with a ground return and a matched termination. When an alternating current wave is transmitted down this line toward the terminated end, about half of the energy is radiated into space. Since there is no reflection at the far end, no return wave exists, and no standing waves appear on the wire, regardless of its length-to-wavelength ratio. Of the energy not radiated, a small amount is dissipated in the wire, and the remainder is dissipated in the terminating resistance.

The long wire is illustrated in Fig. 16.15.1(a), with its horizontal radiation pattern. The radiation pattern shown would be true for any direction at right angles to the wire if the wire were mounted in free space. Usually it is a fraction of a wavelength above the ground, and ground reflections cause most of the energy to be radiated upward so that the vertical pattern would be a single lobe of twice the strength of the horizontal lobes.

This antenna is not often used because it is not very efficient, has a comparatively low gain, and takes up a lot of space. Also, matching the transmitter to the line can be a problem. However, since no standing waves exist, the antenna has no resonances, and as long as the length of the wire lies in the range 2λ to 10λ, its characteristics remain relatively constant for all frequencies in that range. It is thus used as a broadband antenna for low-cost point-to-point communications, especially in the HF band from 3 to 30 MHz. The upward tilt of the pattern lends itself to skywave propagation in this band.



(a)



(b)

Figure 16.15.1 Nonresonant antennas. (a) Long-wire antenna with its horizontal and vertical radiation patterns. (b) Rhombic antenna with its horizontal and vertical radiation patterns.

Rhombic Antenna

The *rhombic antenna* takes its name from its diamond-shaped layout. It is an array of four interconnected long-wire antennas, laid out in the manner shown in Fig. 16.15.1(b). Each of the four legs has the same length and lies in the range 2λ to 10λ. The transmission line feeds one end and transmits an unreflected current wave down each side toward the resistive termination at the far end. The lengths of the sides and the angle φ are interrelated and must be carefully chosen so that the side lobes cancel properly, leaving only a single main lobe lying along the main axis of the rhombus. Again, ground reflections cause the lobe to be tilted upward into the sky, and the amount of tilt is a function of the length of the legs.

The resistive termination is chosen so that no reflections occur, and the antenna is untuned, as is the long-wire antenna. Its frequency range is broad, almost 10 to 1, allowing a single structure to be used over most of the HF bands. It is highly directional and, if the tilt is chosen properly, is ideal for point-to-point skywave propagation.

The feed-point impedance falls in the range from 600 to 800 Ω, allowing direct feed with an open-wire parallel line, and the terminating resistor is in the same range. The angle φ falls between 40° and 75° and the leg length between 2λ and 10λ. The resulting directive gain obtained with the rhombic ranges from 15 to 60. The physical structure is relatively simple and inexpensive and usually takes the form of four poles placed at the apexes of the rhombus and wires supported on tension insulators for the radiators. The feed line is a parallel line and can be any length within reason, although it is usual practice to place the transmitter house near the feed end of the rhombus. For example, a 2λ rhombic for 3 MHz would have diagonals of about 320 m long by about 250 m wide. This antenna is not practical in an urban environment.

Driven Arrays

A *linear array* of antennas consists of a number of basic antenna elements, usually $\frac{1}{2}\lambda$ dipoles, equispaced out along a line referred to as the array axis. By suitably phasing the radiation from each element, the directivity can be altered in a number of ways. To illustrate this, arrays of identical elements operating in the transmitting mode will be considered, as sketched in Fig. 16.16.1(a). It is assumed that each element is fed with currents of equal amplitudes, but with successive phase shifts α. Thus the current to the first element is $I\angle 0$, to the second, $I\angle\alpha$, to the third, $I\angle 2\alpha$, and to the *n*th element, $I\angle(n-1)\alpha$. Figure 16.16.1(b) shows the plan view of the array, and for convenience this is taken to be the equatorial plane. The distance between successive wavefronts is $s \cos \phi$, and therefore the phase lead of element *n* with respect to element $(n-1)$ is $(2\pi/\lambda)s \cos \phi$. Thus the total phase lead of element *n* with respect to $(n-1)$ is

$$\psi = \frac{2\pi}{\lambda} s \cos \phi + \alpha \tag{16.16.1}$$

VHF–UHF Antennas

Discone Omni

The *discone antenna* is designed to radiate an omnidirectional pattern in the horizontal plane, with vertical polarization. It is a broadband antenna with usable characteristics over a frequency range of nearly 10 : 1. It is usually designed to be fed directly from a 50-Ω coaxial line and is mounted directly on the end of that line. The discone is illustrated in Fig. 16.18.1(a).

This type of antenna is ideal for base-station operation for urban mobile communications systems, since it gives a good omnidirectional pattern, is physically very compact and rugged, and is quite inexpensive to construct. Its directional gain along the horizontal plane is comparable to that of the dipole antenna.

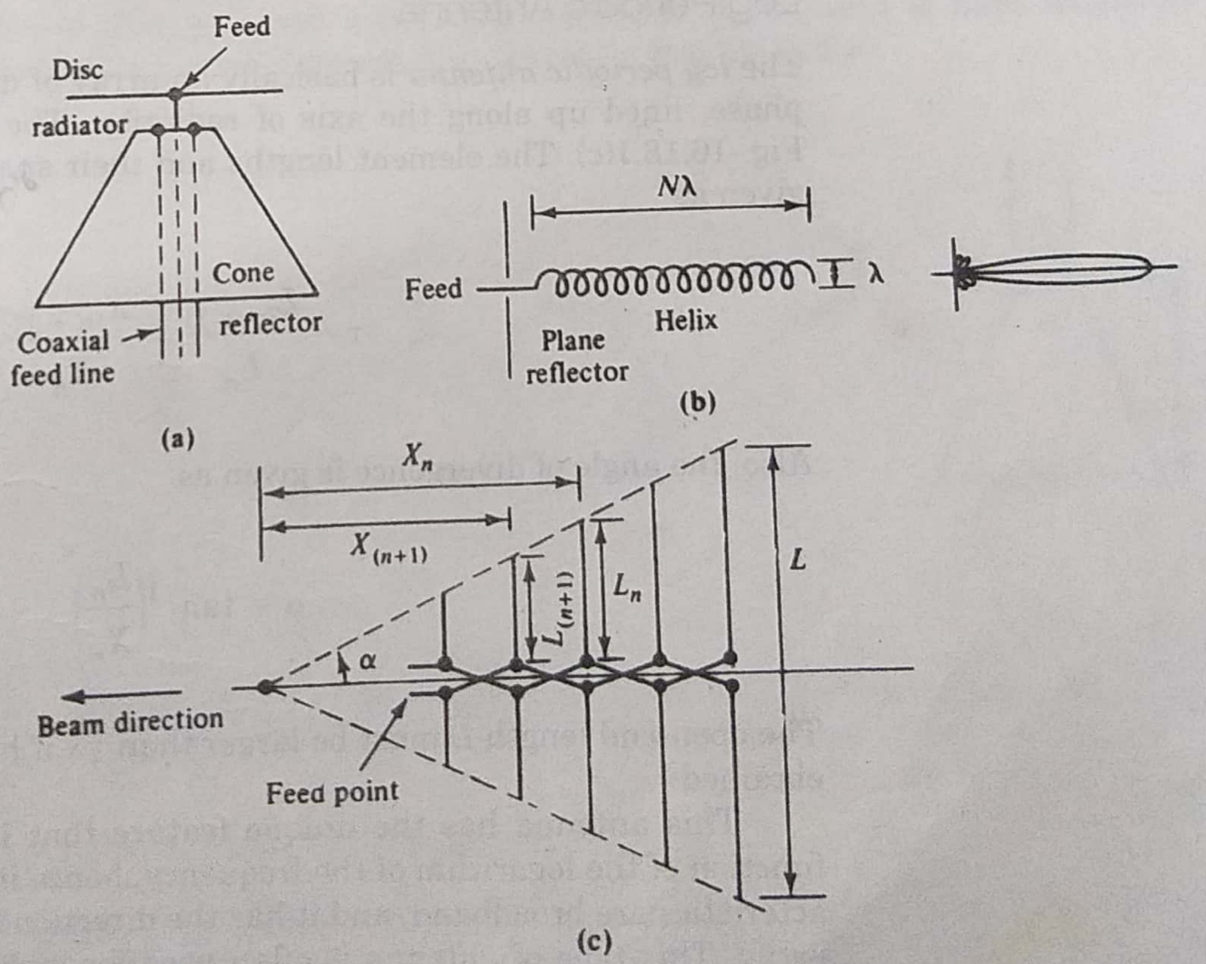


Figure 16.18.1 VHF–UHF antennas. (a) Discone omnidirectional antenna. (b) End-fire helix antenna. (c) Log-periodic dipole array.

Helical Antenna

The radiator element of a *helical antenna* is basically a coil of wire. If the helix diameter is much less than one wavelength, its length less than one wavelength, and it is center-fed, the whole structure will behave very much like a compact dipole antenna, radiating in the "normal" mode. Since the current wave must propagate along the helix conductor, its actual velocity along the axis will be much less than free-space velocity. The velocity will be determined by the diameter and pitch of the helix coil, and the half-wave resonant length of the helix will physically be very much less than the free-space half-wavelength. For most combinations of helix, the polarization is elliptical. This type of antenna is sometimes used in locations where it is not possible to mount a full-sized dipole, such as in urban areas or on rooftops.

If the helix diameter is made approximately one wavelength, and the helix made several wavelengths long and end-fed, the helix radiates in an end-fire mode, producing a narrow beam of circularly polarized waves. The 3-dB beam width obtainable with single helixes is on the order of 15° to 30°. When used in this mode, the radiator is usually end-fed and a plane reflector is placed behind the feed end. The result is a highly directional antenna that is physically compact and that can be easily mounted on a movable table for tracking moving sources. Arrays of end-fire helixes are often used for tracking satellites. These structures are much less expensive than the large parabolas used for radio astronomy and have been very popular with amateur satellite trackers. The end-fire helix is illustrated in Fig. 16.18.1(b).

Log Periodic Antenna

The *log periodic antenna* is basically an array of dipoles, fed with alternating phase, lined up along the axis of radiation. The structure is illustrated in Fig. 16.18.1(c). The element lengths and their spacing all conform to a ratio, given as

$$\tau = \frac{L_{(n+1)}}{L_n} = \frac{X_{(n+1)}}{X_n} \tag{16.18.1}$$

Also, the angle of divergence is given as

$$\alpha = \tan^{-1}\left(\frac{L_n}{X_n}\right) \tag{16.18.2}$$

The open-end length L must be larger than $\frac{1}{2}\lambda$ if high efficiency (90%) is to be obtained.

This antenna has the unique feature that its impedance is a periodic function of the logarithm of the frequency, hence its name. The antenna characteristics are broadband, and it has the directional characteristics of a dipole array. This type of antenna is often used for mobile-base-station operations, where many channels must be handled over a single antenna system with good directive characteristics.

Horns

Radio waves can be radiated directly from the end of a waveguide in the same way as from the end of an open transmission line. The end of the waveguide represents an abrupt transition from the characteristic impedance of the waveguide into that of free space, and the radiation resulting is neither efficient nor very directive. This state of affairs can be improved considerably by flaring out the end of the waveguide to form a hornlike structure. A gradual transition can thus take place as the wave passes from the mouth of the horn.

Narrow-mouthed horns with long flare sections produce sharper beams than shallow, wide-mouthed ones. Also, the wider-mouthed horns tend to produce a wavefront with a distinct curvature, which is undesirable. The ideal would be for the waves to leave the horn with a completely planar wavefront, and to accomplish this a focusing mechanism, such as a curved reflector or a lens, may be used with the horn.

Three types of horns are shown in Fig. 16.19.1. The first is the sectoral horn, which is flared in only one plane [Fig. 16.19.1(a)]; the second is the pyramidal horn, which is flared in both planes [Fig. 16.19.1(b)]. Both of these are used with rectangular waveguides. The third type is conical [Fig. 16.19.1(c)] and is used with a circular waveguide to produce a circularly polarized beam. Horn-type antennas do not provide very high directivity but are of simple, rugged construction. This makes them ideal as primary feed antennas for parabolic reflectors and lenses.

The choice of horn dimensions is dependent on the desired beam angle and directive gain and involves specification of the ratio of flare length to wavelength L/λ and flare angle ϕ, shown in Fig. 16.19.1(d).

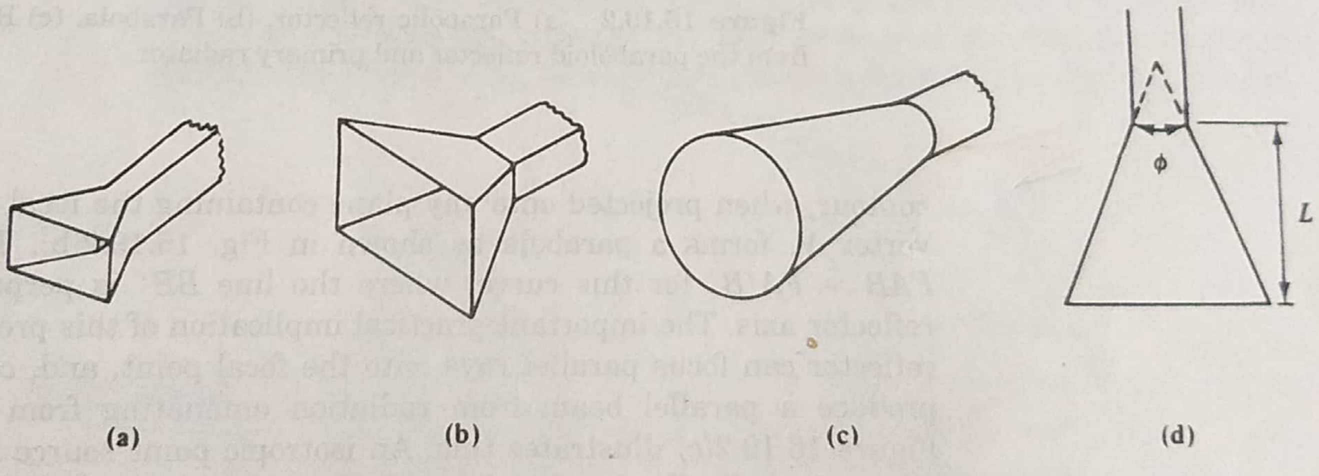


Figure 16.19.1 Microwave horn antennas. (a) Sectoral horn. (b) Pyramidal horn. (c) Conical horn. (d) Horn flare dimensions.

Paraboloidal Reflector Antenna

The most widely used antenna for microwaves is the *paraboloidal reflector antenna,* which consists of a primary antenna such as a dipole or horn situated at the focal point of a paraboloidal reflector, as shown in Fig. 16.19.2. The mouth, or physical aperture, of the reflector is circular, and the reflector

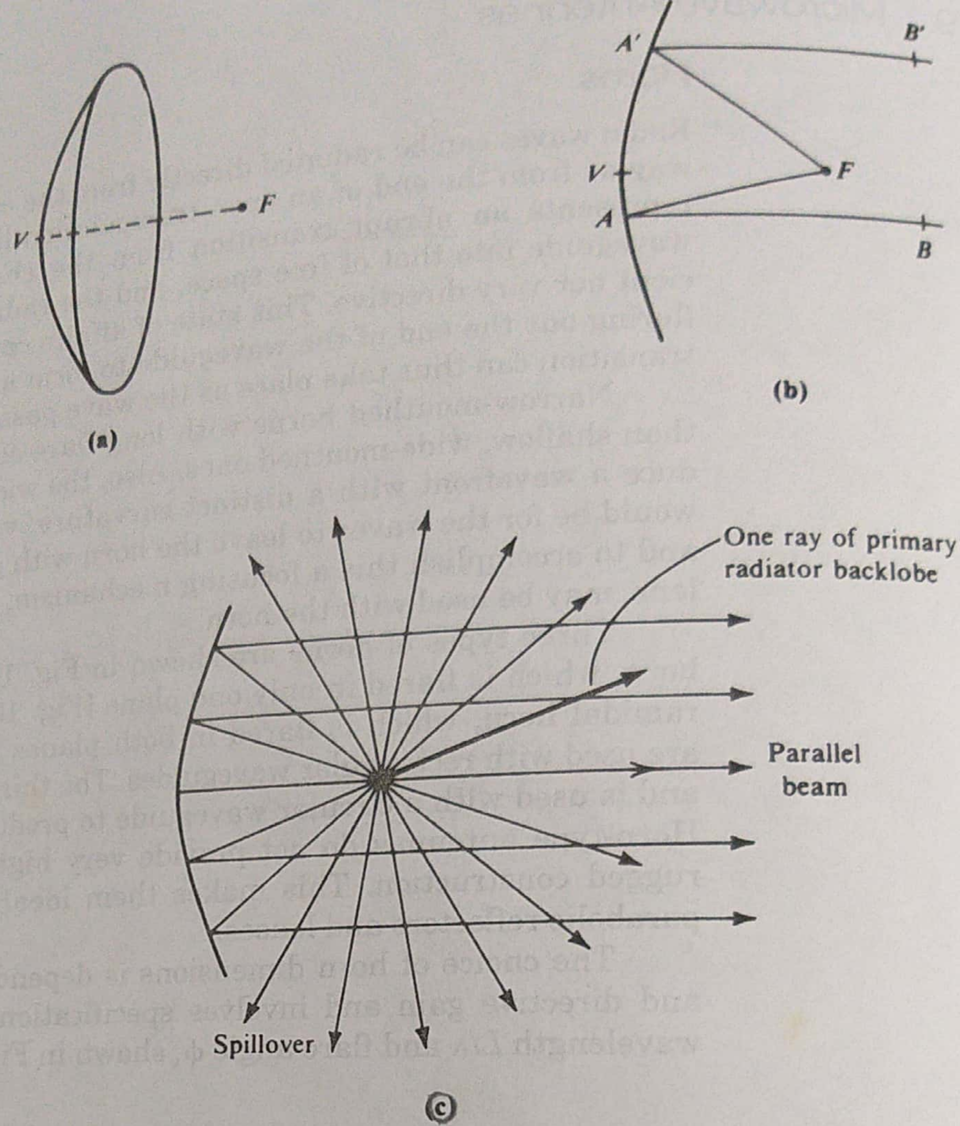


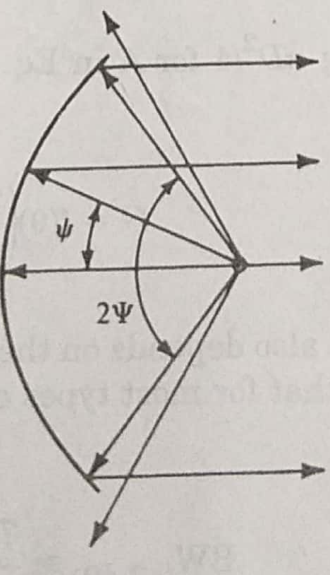
Figure 16.19.2 (a) Parabolic reflector. (b) Parabola. (c) Radiation from the paraboloid reflector and primary radiator.

contour, when projected onto any plane containing the focal point F and the vertex V, forms a parabola as shown in Fig. 16.19.2(b). The path length $FAB = FA'B'$ for this curve, where the line BB' is perpendicular to the reflector axis. The important practical implication of this property is that the reflector can focus parallel rays onto the focal point, and, conversely, it can produce a parallel beam from radiation emanating from the focal point. Figure 16.19.2(c) illustrates this. An isotropic point source is assumed to be situated at the focal point. In addition to the desired parallel beam being shown, it can be seen that some of the rays are not captured by the reflector, and these constitute *spillover*. In the receive mode, spillover increases noise pickup, which can be particularly troublesome in satellite ground stations. Also, some radiation from the primary radiator occurs in the forward direction in addition to the desired parallel beam. This is termed *backlobe* radiation since it is from the backlobe of the primary radiator. Backlobe radiation is undesirable because it can interfere destructively with the reflected beam, and practical radiators are designed to eliminate or minimize this. The isotropic radiator at the focal point will radiate spherical waves, and the paraboloidal reflector converts these to plane waves. Thus, over the aperture of an ideal reflector, the wavefront is of constant amplitude and constant phase.

The directivity of the paraboloidal reflector is a function of the primary antenna directivity and the ratio of focal length to reflector diameter, f/D. This ratio, known as the aperture number, determines the angular aperture of the reflector, 2Ψ [Fig. 16.19.3(a)], which in turn determines how much of the primary radiation is intercepted by the reflector. Assuming that radiation from the primary antenna is circularly symmetric about the reflector axis $(F - V)$ and is confined to angles ψ in the range $-\pi/2 < \psi < \pi/2$, it is found that the effective area is given by

$$A_{\text{eff}} = AI(\theta) \tag{16.19.1}$$

where $A = \pi D^2/4$ is the physical area of the reflector aperture, and $I(\theta)$ is a function, termed the *aperture efficiency* (or *illumination efficiency*), which takes into account both the radiation pattern of the primary radiator and the effect of the angular aperture. With the focal point outside the reflector, as shown in Fig. 16.19.3(a) (which requires $f/D > 1/4$), the primary radiation at the perimeter of the reflector will not be much reduced from that at the center, and the reflector illumination approaches a uniform value. This increases the aperture efficiency, but at the expense of spillover occurring. Making f/D too large increases spillover to the extent that aperture efficiency then decreases. Reducing f/D to less than 1/4 places the focal point inside the reflector, as shown in Fig. 16.19.3(b). Here, no spillover occurs, but the illumination of the reflector tapers from a maximum at the center to zero within the reflector region. This nonuniform illumination tends to reduce aperture efficiency. Also, placing the primary antenna too close to the reflector results in the reflector affecting the primary antenna impedance and radiation pattern,



(a)

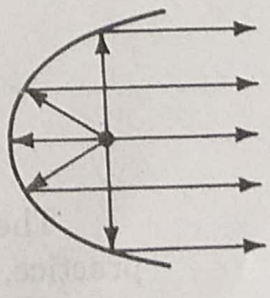

(b)

Figure 16.19.3 (a) Focal point outside the reflector. (b) Focal point inside the reflector.

which is difficult to take into account. It can be shown that the aperture efficiency peaks at about 80%, with the angular aperture ranging from about 40° to 70° depending on the primary radiation pattern. The relationship between aperture number and angular aperture is

$$\frac{f}{D} = 0.25 \cot\left(\frac{\Psi}{2}\right) \tag{16.19.2}$$

Typically, for an angular aperture of 55°, the aperture number is

$$\frac{f}{D} = 0.25 \times 1.92 = 0.48$$

This shows that the focal point should lie outside the mouth of the reflector, since f/D is then greater than 1/4. Satisfactory results are obtained in practice if the main lobe of the primary antenna intercepts the perimeter of the reflector at the −9 to −10 dB level as shown in Fig. 16.19.4.

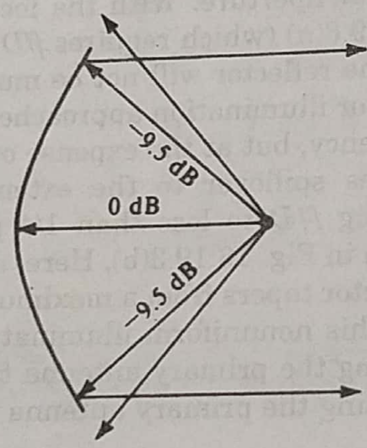


Figure 16.19.4 Edge illumination from primary antenna is −9 to −10 dB below that at the center.

On substituting $\pi D^2/4$ for A in Eq. (16.19.1) and using Eq. (16.8.3) for gain, we get

$$G = I(\theta)\left(\frac{\pi D}{\lambda}\right)^2 \tag{16.19.3}$$

The beamwidth also depends on the primary radiator and its position. In practice, it is found that for most types of feed the −3-dB beamwidth is given approximately by

$$\text{BW}_{(-3\text{-dB})} \cong \frac{70\lambda}{D} \text{ degrees} \tag{16.19.4}$$

and the beamwidth between nulls by

$$\text{nulls BW} = 2[\text{BW}_{(-3\text{-dB})}]$$

$$= \frac{140\lambda}{D} \text{ degrees} \qquad (16.19.5)$$

EXAMPLE 16.19.1

Find the directivity, beamwidth, and effective area for a paraboloidal reflector antenna for which the reflector diameter is 6 m and the illumination efficiency is 0.65. The frequency of operation is 10 GHz.

SOLUTION

$$\lambda = \frac{c}{f} = \frac{300 \times 10^6}{10 \times 10^9} = 0.03 \text{ m} = 3 \text{ cm}$$

$$A = \frac{\pi D^2}{4} = \frac{3.14 \times 6^2}{4} = 28.26 \text{ m}^2$$

$$A_{\text{eff}} = 0.65A = 18.4 \text{ m}^2$$

$$D_0 = \frac{4\pi}{\lambda^2} A_{\text{eff}} = 257{,}000 \text{ (54.1 dB)}$$

$$\text{BW}_{(-3\text{-dB})} = \frac{70\lambda}{D} = \frac{70 \times 0.03}{6} = 0.35°$$

$$\text{BW}_{(\text{null})} = 2 \times 0.35 = \mathbf{0.70°}$$

Variations on the Parabolic Reflector

The *parabolic reflector* is a favorite antenna for fixed point-to-point microwave communications systems. It is relatively simple in construction, and unless large in size, it is quite inexpensive. Huge steerable parabolic dishes have been built for use with the radio telescopes, up to 200 ft in diameter, and mounted on a movable turret that allows rotation in both the horizontal and vertical directions to allow the tracking of moving targets such as satellites and radio stars.

Antennas used for radioastronomy must utilize all their area for reception to get the highest efficiency and the lowest noise figure. Special feed systems are used so that the feed antenna is reduced in size or physically located out of the path of the incoming radiation. Two types of feed are shown in Fig. 16.19.5. The first of these uses a dipole antenna, which normally radiates outside the parabola as well as onto it, but has a spherical reflector placed directly behind the dipole to prevent direct radiation. The backlobe radiation is reflected back at the parabola and is added to the main portion of the radiation. Some tuning is necessary since the reflector position is different for different frequencies.

The second method is known as the *Cassegrain feed system* [Fig. 16.19.5(b)]. The horn feed antenna, the paraboloid reflector, and the hyperboloid subreflector have a common axis of symmetry as shown, and the virtual

An electric current in a wire is always surrounded by a magnetic field. When the current is alternating, the free electric charges in the wire are accelerated, which gives rise to an alternating electromagnetic field, which travels away from the wire in the form of an electromagnetic wave. (An analogy may be made by considering a semirigid sheet being moved at constant velocity, which results in a steady air displacement. If the sheet is vibrated, which in effect imparts acceleration to some areas of it, a sound wave will be generated that travels through the air away from the sheet.)

The total field originating from an alternating current in a wire is complicated, consisting of (1) an electric field component that lags the current by 90° and that decreases in amplitude as the cube of the distance; (2) an electromagnetic field (a combined electric and magnetic field) that is in phase with the current and that decreases in amplitude as the square of the distance; and (3) an electromagnetic field that leads the current by 90° and that decreases in amplitude directly as the distance increases. Only the latter electromagnetic field reaches the receiver in a normal radio communications system, where it appears to the receiving antenna as a plane transverse electromagnetic (TEM) wave. The basic properties of a TEM wave are discussed in Appendix B. A useful rule of thumb is that for antennas for which the largest dimension D is very much greater than the wavelength being radiated ($D \gg \lambda$), the far-field zone becomes the only significant one for distances d greater than $2D^2/\lambda$:

$$d \gtrsim \frac{2D^2}{\lambda} \qquad (16.4.1)$$

Polarization

In the far-field zone, the *polarization* of the wave is defined by the direction of the electric field vector in relation to the direction of propagation. *Linear* polarization is when the electric vector remains in the same plane, as shown in Fig. 16.5.1(a). A linear polarized wave that is propagated across the earth's surface is said to be *vertically polarized* when the electric field vector is vertical and *horizontally polarized* when it is parallel to the earth's surface. For

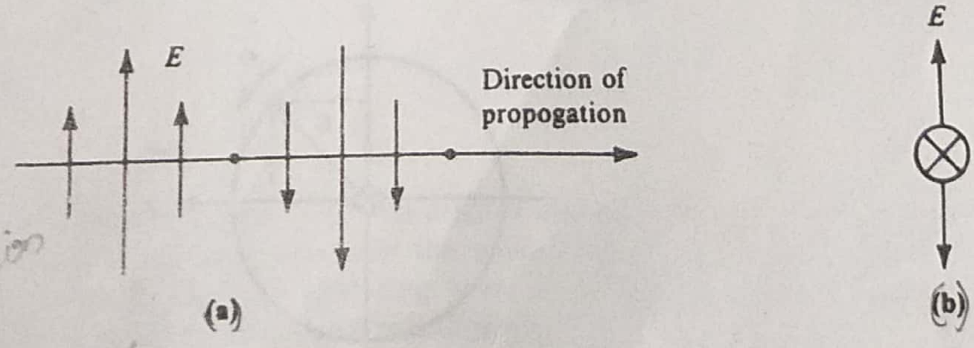


Figure 16.5.1 Linear polarization as viewed (a) on the axis of propagation and (b) along the direction of propagation.

example, in North America, television transmissions are horizontally polarized, and it will be observed that receiving antennas are also horizontally mounted, whereas in the United Kingdom vertical polarization is used, and there antennas are mounted vertically.

In certain situations the electric vector may rotate about the line of propagation. This can be caused, for example, by the interaction of the wave with the earth's magnetic field in the F_2 layer of the ionosphere. Rotation of the electric vector can also be produced by the type of antenna used, and this effect is put to good use in satellite communications, as described in Chapter 19. The path traced out by the tip of the electric vector may be an ellipse, as illustrated in Fig. 16.5.2, in which case it is referred to as *elliptical polarization.* If the rotation is in a clockwise direction when looking along the direction of propagation, the polarization is referred to as *right-handed;* if it is anticlockwise, it is called *left-handed.* In Fig. 16.5.2(a) the direction of propagation is into the paper, so the polarization is right-handed. A special case of elliptical polarization is *circular polarization,* as illustrated in Fig. 16.5.3, and both right-handed and left-handed circular polarization are used in satellite communications systems as described later. Linear polarization can also be considered to be a special case of elliptical polarization. As shown in Fig. 16.5.2, elliptical polarization can be resolved into two linear vectors, E_x and E_y. Linear polarization results when one of these components is zero.

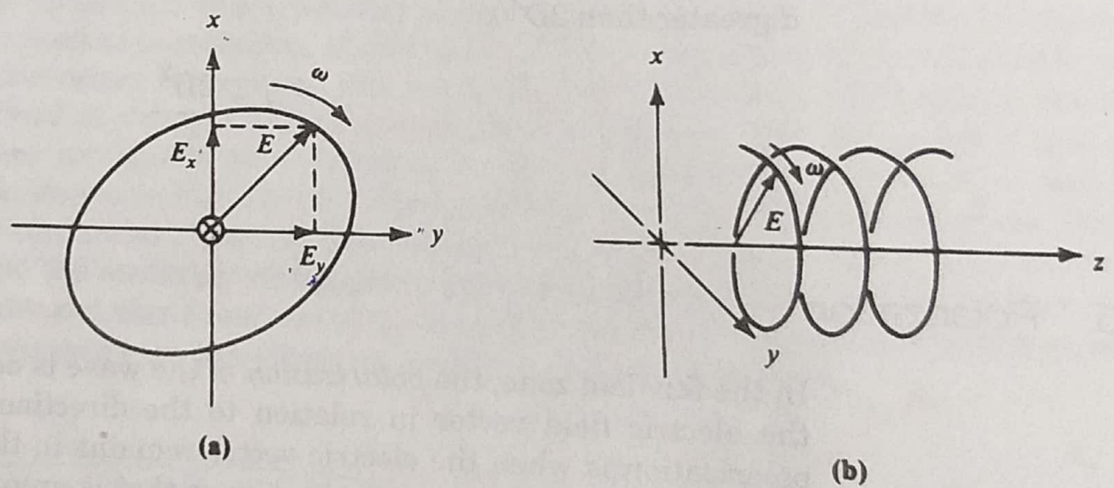


Figure 16.5.2 Elliptical polarization viewed (a) along the direction of propagation and (b) on the axis of propagation.

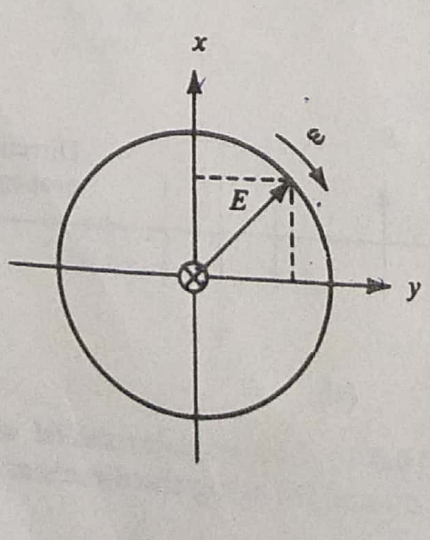


Figure 16.5.3 Circular propagation.

To receive a maximum signal, the polarization of the receiving antenna must be the same as that of the transmitting antenna, which is defined to be the same as that of the transmitted wave. For example, a wire dipole antenna, illustrated in Fig. 16.5.4(a), will radiate a linear polarized wave. A similar receiving dipole must be oriented parallel to the electric vector for maximum reception. If it is at some angle ψ, as illustrated in Fig. 16.5.4(b), then only the component of the electric field parallel to the receiving antenna will induce a signal in it. This component is $E \cos \psi$, and therefore the polarization loss factor is

$$\text{plf} = \cos \psi \tag{16.5.1}$$

A similar situation can exist with an aperture antenna as illustrated in Fig. 16.5.4(c). The angle ψ is the angle between the induced field in the aperture and the incoming electric field, and the polarization loss factor is also cos ψ in this case. In both cases the direction of propagation is normal to the plane of the antenna.

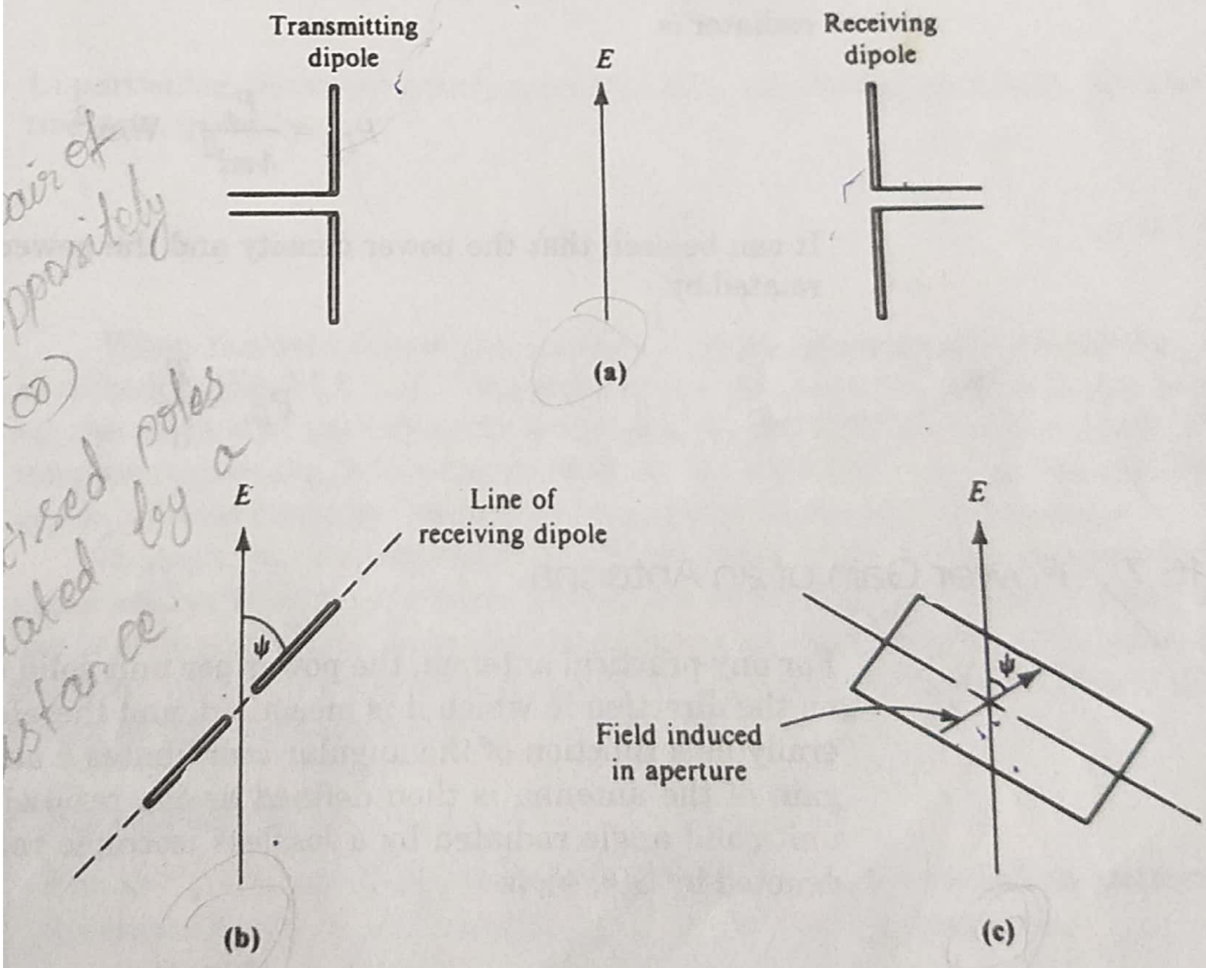


Figure 16.5.4 (a) Two dipoles aligned with the same polarization. (b) Receiving dipole in the same plane as E but with polarization misaligned. (c) Incoming wave E in same plane as aperture, but aperture polarization misaligned.

Isotropic Radiator

The word *isotropic* means "equally in all directions," so an isotropic radiator is one that radiates equally in all directions. A star is an example of an isotropic radiator of electromagnetic energy, but on a more practical level, all real antennas radiate better in some directions than others and cannot be isotropic. However, the concept of an isotropic radiator is a very useful one and provides a standard to which real antennas can be compared. Furthermore, since this is a hypothetical radiator, it may be assumed lossless; that is, its efficiency is unity. Let P_s represent the power input to a lossless isotropic radiator. Then since its efficiency is unity, this is also the power radiated. Consider this antenna at the center of the sphere shown in Fig. 16.3.1. Then, since any sphere has a solid angle of 4π steradians at its center, the power per unit solid angle is

$$P_i = \frac{P_s}{4\pi} \quad \text{W/sr} \tag{16.6.1}$$

This quantity is used as a standard to which real antennas can be compared. Another useful quantity is the power density. The surface area of a sphere of radius d is $4\pi d^2$, and therefore the power density for the lossless isotropic radiator is

$$P_{Di} = \frac{P_s}{4\pi d^2} \quad \text{W/m}^2 \tag{16.6.2}$$

It can be seen that the power density and the power per unit solid angle are related by

$$P_{Di} = \frac{P_i}{d^2} \tag{16.6.3}$$

Power Gain of an Antenna

For any practical antenna, the power per unit solid angle will vary depending on the direction in which it is measured, and therefore it may be written generally as a function of the angular coordinates θ and ϕ as $P(\theta, \phi)$. The power gain of the antenna is then defined as the ratio of $P(\theta, \phi)$ to the power per unit solid angle radiated by a lossless isotropic radiator. The gain function, denoted by $G(\theta, \phi)$, is

$$G(\theta, \phi) = \frac{P(\theta, \phi)}{P_i} = \frac{4\pi P(\theta, \phi)}{P_s} \tag{16.7.1}$$

The gain function is a very important antenna characteristic that can be measured or, in some cases, calculated, and some examples will be given later.

For most antennas, the gain function shows a well-defined maximum, which will be denoted by G_M, and the radiation pattern of the antenna is

$$g(\theta, \phi) = \frac{G(\theta, \phi)}{G_M} \qquad (16.7.2)$$

The radiation pattern is seen to be simply the gain function normalized to its maximum value. The maximum value G_M is referred to as the *gain* of the antenna, but this is only a gain in the sense that the antenna concentrates or focuses the power in the maximum direction. It does not increase the total power radiated.

Closely associated with the power gain is the *directive gain* of the antenna. This is the ratio of $P(\theta, \phi)$ to the average power per unit solid angle radiated by the *actual* antenna and is denoted by $D(\theta, \phi)$. The average power per unit solid angle is $\eta_A P_s/4\pi$, where η_A is the antenna efficiency and P_s is the power input, as before. Thus the average is seen to be equal to $\eta_A P_i$, and therefore the directivity is related to power gain by

$$D(\theta, \phi) = \frac{G(\theta, \phi)}{\eta_A} \qquad (16.7.3)$$

In particular, the maximum value of $D(\theta, \phi)$ is termed the *directivity*, or *directive gain*, given by

$$D_M = \frac{G_M}{\eta_A} \qquad (16.7.4)$$

When the gain function is plotted, a three-dimensional plot results, as sketched in Fig. 16.7.1(a). The length of the line from the origin to any point on the surface of the figure gives the gain in the direction of the point. The maximum gain G_M is shown, as well as the gain $G(\theta_1, \phi_1)$ in the direction (θ_1, ϕ_1). Minor lobes, as indicated by G_2 and G_3, also occur in general.

In practice, two-dimensional plots are often used, one for the equatorial plane and one for the meridian plane, the function $g(\theta, \phi)$ usually being the one that is plotted. In the equatorial plane this is denoted by $g(\phi)$, since θ is constant, and in the meridian plane by $g(\theta)$, since ϕ is constant. This is illustrated in the following example.

EXAMPLE 16.7.1

For the Hertzian dipole (to be described later), the radiation pattern is described by $g(\theta) = \sin^2\theta$ and $g(\phi) = 1$. Plot the polar diagrams.

SOLUTION The polar diagrams are plotted in Fig. 16.7.1(b) and (c).

The −3-*dB beamwidth* of an antenna is the angle subtended at the center of the polar diagram by the −3-dB gain lines. This is illustrated in Fig. 16.7.2, where the beamwidth is

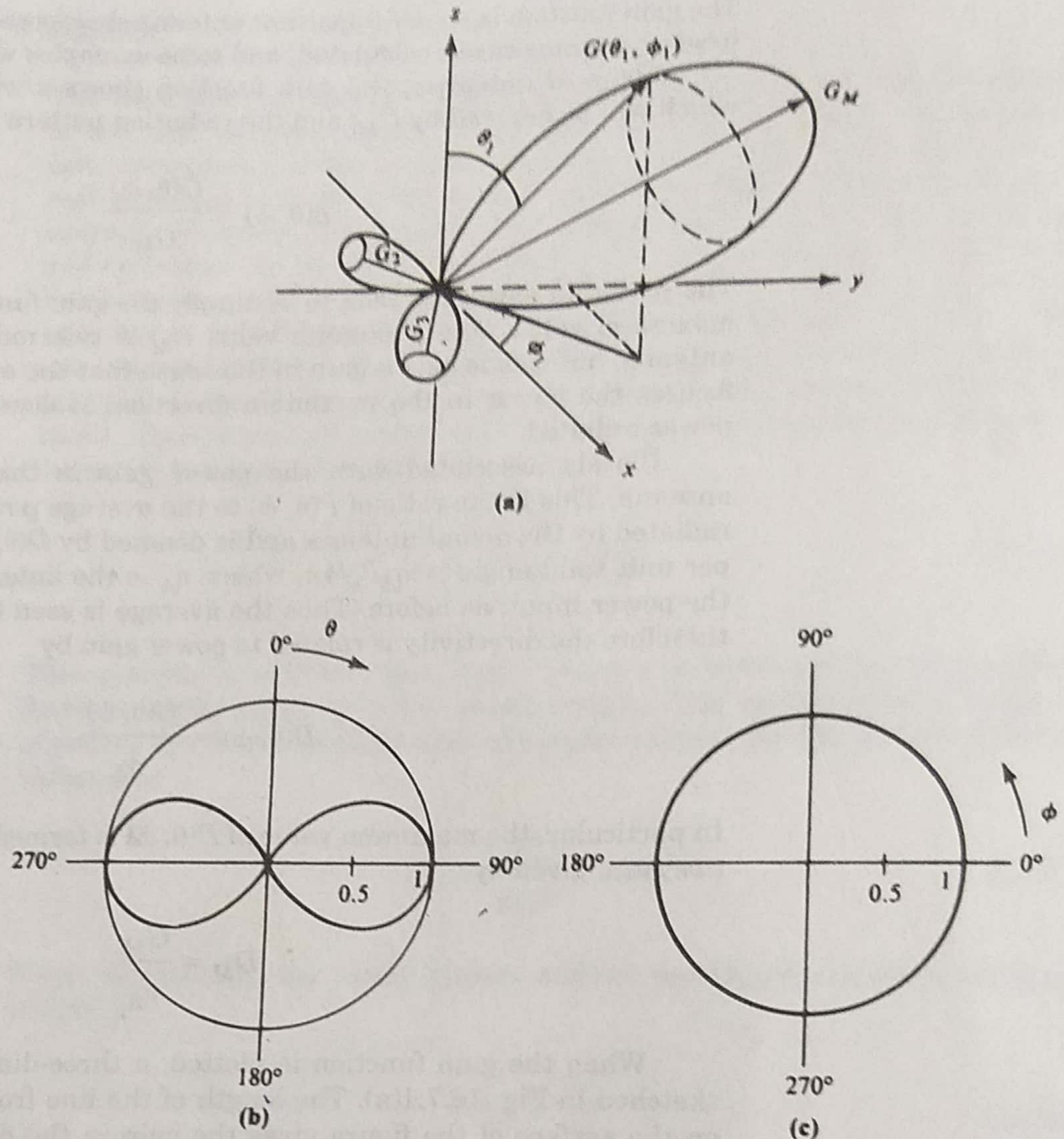


Figure 16.7.1 (a) Gain function $G(\theta, \phi)$. (b) Polar diagram of function $g(\theta)$. (c) Polar diagram for function $g(\phi)$ for Example 16.7.1.

$$\theta_3 = \theta_2 - \theta_1 \qquad (16.7.5)$$

EXAMPLE 16.7.2

Determine the −3-dB beamwidth for the Hertzian dipole of Example 16.7.1.

SOLUTION $g(\theta_1) = \sin^2 \theta_1 = 0.5$, giving $\theta_1 = 45°$. Also, $g(\theta_2) = \sin^2 \theta_2 = 0.5$, giving $\theta_2 = 135°$. Therefore,

$$\theta_3 = \theta_2 - \theta_1 = 135° - 45° = \mathbf{90°}$$

It will be observed from this example that the beamwidth applies only to the meridian plane for this antenna, since the equatorial plane polar diagram is a circle.

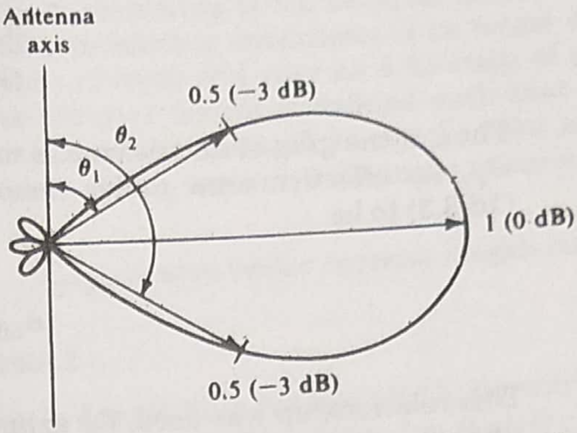


Figure 16.7.2 −3-dB Beamwidth of an antenna.

In certain cases the beamwidth may be specified for levels other than −3 dB, other common values being −10 and −60 dB. Obviously the beamwidth level must be specified along with the beamwidth.

Effective Area of an Antenna

A receiving antenna may be thought of as having an effective area that collects electromagnetic energy from the incident wave, rather as a solar collector collects energy from sunlight. Assuming that the antenna is in the far-field zone of the radiated wave, the wave incident on it will be a plane TEM wave having a power density of P_D W/m^2 of wavefront. Let the receiving antenna be at the center of a spherical coordinate system, and let the incoming wave direction be specified by the angular coordinates (θ, ϕ) with reference to the antenna. The power delivered to a matched load (receiver) will be a function of direction, and this is taken into account by making the effective area a function of direction; that is, $A = A(\theta, \phi)$. Thus, if P_R is the power delivered to a matched load,

$$P_R = P_D A(\theta, \phi) \tag{16.8.1}$$

Equation (16.8.1) serves as a defining equation for effective area. The effective area will have some maximum value A_{eff}, called the *effective area of the antenna*, just as the maximum power gain is called the gain of the antenna. As a result of the reciprocity theorem, the effective area normalized to its maximum value has the same functional form as the normalized power gain [Eq. (16.7.2)]; that is,

$$\frac{A(\theta, \phi)}{A_{\text{eff}}} = g(\theta, \phi) \tag{16.8.2}$$

It follows that A_{eff} is proportional to G_M. It can also be shown from the reciprocity theorem that the constant of proportionality is the *same for all antennas:* $\lambda^2/4\pi$. Thus

$$\frac{A_{\text{eff}}}{G_M} = \frac{\lambda^2}{4\pi} \tag{16.8.3}$$

Thus, if the gain of an antenna is measured under transmitting conditions as G_T, its effective area under receiving conditions can be found from Eq. (16.8.3) to be

$$A_{\text{eff}} = \frac{\lambda^2 G_T}{4\pi} \tag{16.8.4}$$

This relationship was used, for example, in deriving Eq. (15.2.5) for free-space transmission. Note that G_T takes into account antenna efficiency, and therefore so does the effective area. Often, in theoretical calculations, the directivity D_T is used in Eq. (16.8.4) instead of G_T, and this will give a higher value of effective area since directivity excludes the antenna efficiency.

Another factor that can reduce the effective area is the mismatch factor, given by Eq. (16.2.7). The effective area is reduced directly by this factor, and, of course, if the antenna is matched to the line, no reduction occurs.

Previously, the effective area was shown to be a function of the angular coordinates θ and ϕ. Defined this way, $A(\theta, \phi)$ automatically takes into account any loss resulting from polarization misalignment. However, it is usually the maximum value A_{eff} of $A(\theta, \phi)$ that is known, as well as the polarization loss factor plf, as given by Eq. (16.5.1). Since the plf is defined for electric field strength, and A_{eff} for power, the reduction in A_{eff} as a result of polarization misalignment is $(\text{plf})^2$.

Effective Length of an Antenna

Although the concept of effective area can be used with any antenna, it is particularily useful with microwave antennas. At lower frequencies, where the physical structure of the antenna is of the form of a linear conductor or an array of conductors, an analogous concept, the *effective length*, proves to be more useful.

For a receiving antenna, the open circuit emf appearing at the terminals is V_A, the Thévenin equivalent voltage, as shown in Fig. 16.2.1(c). Now, this is produced by a wave having an electric field strength of E V/m sweeping over the antenna, and therefore an effective length ℓ_{eff} can be defined by

$$V_A = E\ell_{\text{eff}} \tag{16.9.1}$$

The effective length ℓ_{eff} as defined by Eq. (16.9.1) is the maximum value. The effective length will in general be a function of θ and ϕ, just as the effective area is in general, and with ℓ_{eff} it is to be understood that the antenna is oriented for maximum induced emf, so, for example, the polarization loss factor would be unity.

For the transmitting antenna, effective length is defined in a different manner, but it can be shown by the use of the reciprocity theorem that the effective length for transmitting is the same as that for receiving. The definition of ℓ_{eff} under transmitting conditions is in terms of the current distribution. The antenna current will vary as a function of physical length along the antenna. The effective length is defined such that the product of input terminal current and effective length is equal to the area under the actual current–length curve. Let I_0 represent the input terminal current; then

$$I_0 \ell_{eff} = \text{area under current–length curve} \tag{16.9.2}$$

EXAMPLE 16.9.1

For the $\frac{1}{2}\lambda$ dipole described in Section 16.11, the current–length curve may be assumed to be $I = I_0 \cos \beta\ell$, where $\ell = 0$ at the input terminals. Find the effective length.

SOLUTION The physical length of the antenna is $\lambda/2$, and the average current for the cosine distribution is

$$I_{av} = \frac{2}{\pi} I_0$$

Thus

$$\text{area} = I_{av} \times \text{physical length}$$

$$= \frac{2}{\pi} I_0 \frac{\lambda}{2} = \frac{I_0 \lambda}{\pi}$$

Hence, from Eq. (16.9.2),

$$\ell_{eff} = \frac{\lambda}{\pi}$$

For low- and medium-frequency antennas that are mounted vertically from the earth's surface, the effective length is usually referred to as the *effective height* h_{eff}. This is directly related to the physical height. Effective height must not be confused with the physical heights h_T and h_R introduced in Section 15.3. For example a $\frac{1}{2}\lambda$ dipole may be mounted at some mast height h above the ground, but its effective length is *always* λ/π.

Hertzian Dipole

The Hertzian dipole is a short linear antenna that, when radiating, is assumed to carry uniform current along its length. Such an antenna cannot be realized in practice, but longer antennas can be assumed to be made up of a number of Hertzian dipoles connected in series. The radiation properties of the Hertzian dipole are readily calculated. This is useful in itself in that it helps to illustrate the general properties discussed in the previous sections, but also the properties of longer antennas can often be deduced by superimposing the results of the chain of Hertzian dipoles making up the longer antenna.