2. ISOCOST CURVES

Having studied the nature of isoquants which represent the output possibilities of a firm from a given combination of two inputs, we pass on to the prices of the inputs as represented on the isoquant map by the isocost curves. These curves are also known as *outlay lines, price lines, input-price lines, factor-cost lines, constant-outlay lines, etc.* Each isocost curve represents the different combinations of two inputs that a firm can buy for a given sum of money at the given price of each input.

Figure, 29.8 (A) shows three isocost curves, each represents a total outlay of 50, 75 and 100 respectively. The firm can hire *OC* of capital or *OD* of labour with Rs.

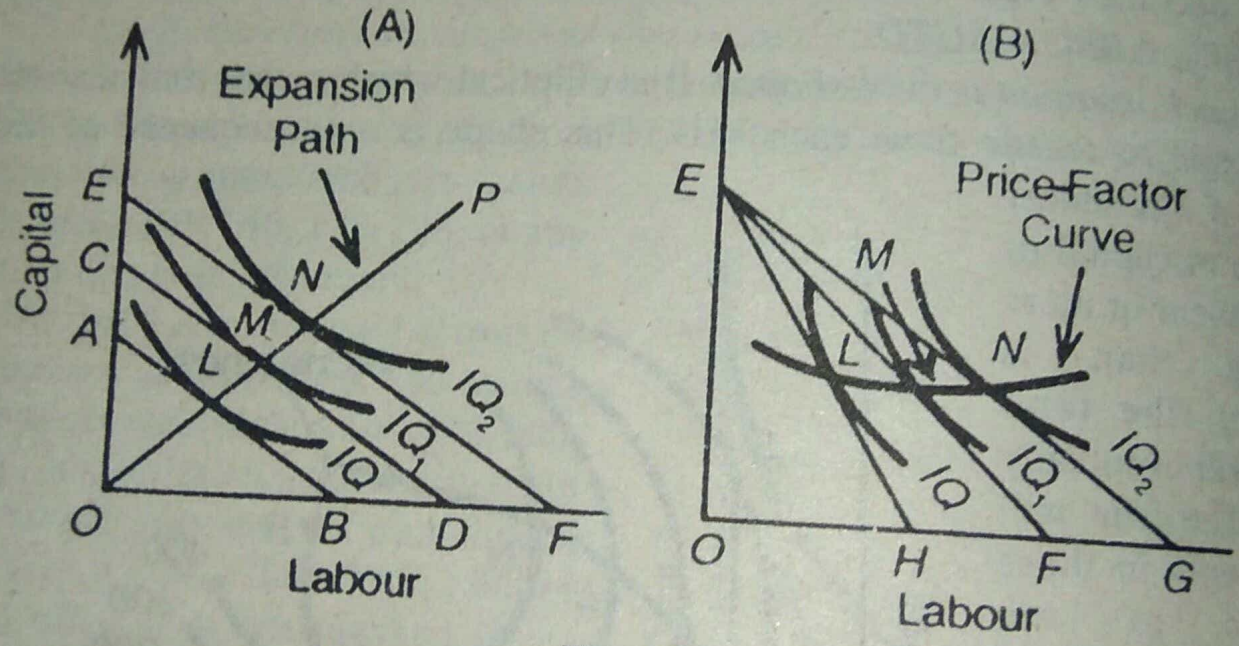

Fig. 29.8

75. *OC* is 2/3 *of OD* which means that the price of a unit of labour is 1½ times less than that of a unit of capital. The line *CD* represents the price ratio of capital and labour. Prices of factors remaining the same, if the total outlay is raised, the isocost curve will shift upward to the right as *EF* parallel to *CD*, and if the total outlay is reduced it will shift downwards to the left as *AB*. The isocosts are straight lines because factor prices remain the same whatever the outlay of the firm on the two factors. *The isocost curves represent the locus of all combinations of the two input factors which result in the same total cost.* If the unit cost of labour *(L)* is *w* and the unit cost of capital (*C*) is *r*, then the total cost: $TC = wL + rC$. The slope of the isocost line is the ratio of prices of labour and capital *i.e., w/r.*

The point where the isocost line is tangent to an isoquant represents the least cost combination of the two factors for producing a given output. If all points of tangency like *LMN* are joined by a line, it is known as an *output- factor curve* or *least-outlay curve* or *the expansion path* of a firm. It shows how the proportions of the two factors used might be changed as the firm expands. For example, in Figure 29.8 (A) the proportions of capital and labour used to produce 200 (IQ_1) units of the product are different from the proportions of these factors used to produce 300 (IQ_2) units or 100 (OQ) units at the lowest Cost[4].

Like the price-income line in the indifference curve analysis, a relative cheapening of one of the factors to that of another will extend the isocost line to the right. If one of the factors becomes relatively dearer, the isocost line will contract inward to the left. Given the price of capital, if the price of labour falls, the isocost line *EF* in Panel *(B)* will extend to the right as *EG* and if the price of labour rises, the isocost line *EF* will contract inward to the left as *EH*. if the equilibrium points *L, M,* and *N* are joined by a line, it is called the *price-factor curve.*

Chapter 29

Laws of Returns: The Isoquant-Isocost Approach

In the preceding chapter, the various production functions were explained in terms of the traditional analysis. This chapter explains them with the help of the isoquant-isocost approach. The technique involved here is similar to the indifference curve technique used in consumption theory.

1. ISOQUANTS

An isoquant (isoproduct) is a curve on which the various combinations of labour and capital show the same output. "An isoproduct curve is a curve along which the maximum achievable rate of production is constant."[1] It is also known as a production indifference curve or a constant product curve.[2] Just as an indifference curve shows the various combinations of any two commodities that give the consumer the same amount of satisfaction (iso-utility), similarly an isoquant indicates the various combinations of two factors of production which give the producer the same level of output per unit of time. Table 29.1 shows a hypothetical isoquant schedule of a firm producing 100 units of a good.

TABLE 29.1 : Isoquant Schedule

Combination	*Units of Capital*	*Units of Labour*	*Total Output (in units)*
A	*9*	*5*	*100*
B	*6*	*10*	*100*
C	*4*	*15*	*100*
D	*3*	*20*	*100*

This Table 29.1 is illustrated on Figure 29.1 where labour units are measured along the *X*-axis and capital units on the *Y*-axis. The first, second, third and the fourth combinations are shown as *A*, *B*, *C* and *D* respectively. Connect all these points and we have a curve *IQ*. This is an isoquant. The firm can produce 100 units of output at point *A* on this curve by having a combination of 9 units of capital and 5 units of labour. Similarly, point *B* shows a combination of 6 units of capital and 10 units of labour; point *C*,4 units of capital and 15 units of labour; and point *D*, a combination of 3 units of capital and 20 units of labour to yield the same output of 100 units.

An isoquant map shows a number of isoquants representing different amounts of output. In Figure 29.1, curve *IQ*, IQ_1 and IQ_2 show an isoquant map.

1. K.J.Cohen and R.M.Cyert,*Theory of the Firm*, p.113.
2. Iso means equal, so it is an *equal-quantity* or *equal-product (iso-product) curve*.

Starting from the curve IQ which yields 100 units of product, the curve IQ_1 shows 200 units and the IQ_2 curve 300 units of the product which can be produced with altogether different combinations of the two factors.[3]

1. (A) ISOQUANTS Vs. INDIFFERENCE CURVES

An isoquant is analogous to an indifference curve in more than one way. In it, two factors (capital and labour) replace two commodities of consumption. An isoquant shows equal level of product while an indifference curve shows equal level of satisfaction at all points. The properties of isoquants, as we shall study below, are exactly similar to those of indifference curves. However, there are *certain differences* between isoquants and indifference curves. *Firstly*, an indifference curve represents satisfaction which cannot be measured in physical units. In the case of an isoquant the product can be measured in physical units. *Secondly*, on an indifference map one can only say that a higher indifference curve gives more satisfaction than a lower one, but it cannot be said how much more or less satisfaction is being derived from one indifference curve as compared to the other, whereas one can easily tell by how much output is greater on a higher isoquant in comparison with a lower isoquant. In Figure 29.1 output on the curve IQ_1 is double, and on the IQ_2 treble than on the curve IQ. *Lastly*, since satisfaction on indifference curves cannot be measured in physical units, they are given arbitrary numbers 1, 2, 3, 4, etc. The isoquants have an added advantage over the former because they can be labelled in physical units, as 100, 200, 300, etc. in Figure 29.1, to indicate the output level to which each curve corresponds.

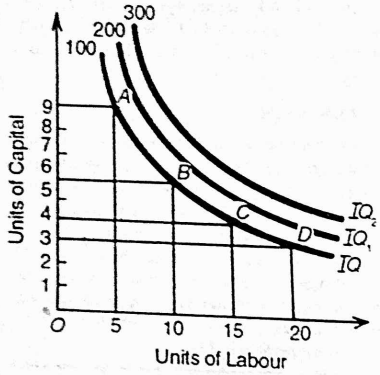

Fig. 29.1

1. (B) PROPERTIES OF ISOQUANTS

Isoquants possess certain properties which are similar to those of indifference curves.

(1) *Isoquants are negatively inclined.* If they do not have a negative slope, certain logical absurdities follow. If the isoquant slopes upward to the right, it implies that both capital and labour increase but they produce the same output. In Figure 29.2 (A), combination B on the IQ curve having a Larger amount of both

3. The isoquant analysis is also known as *one-output two-inputs production function.*

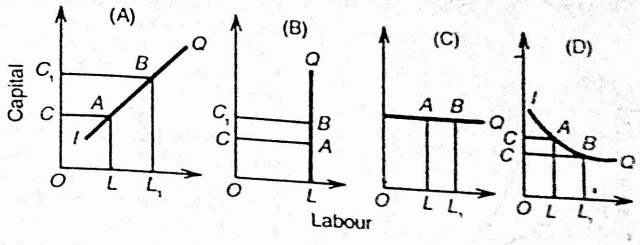

Fig. 29.2

capital and labour ($OC_1 + OL_1 > OC + OL$) will yield more output than before. Therefore, point A and B on the IQ curve cannot be of equal product.

Suppose the isoquant is vertical as shown in Figure 29.2 *(B)*, which implies a given amount of labour is combined with different units of capital. Since OL of labour and OC_1 of capital will produce a larger amount than produced by OL of labour and OC of capital, the isoquant IQ cannot be a constant product curve.

Take Figure 29.2 *(C)* where the isoquant is horizontal which means combining more of labour with the same quantity of capital. Here OI of capital and OL_1 of labour will produce a larger or smaller amount than produced by the combination OI of capital and OL of labour. Therefore, a horizontal isoquant cannot be an equal product curve.

Thus it is clear that an isoquant must slope downward to the right as shown in Figure 29.2 (D) where points A and B on the IQ curve are of equal quantity. As the amount of capital decreases from OC to OC_1 that of labour increases from OL to OL_1 so that output remains constant

(2) An *Isoquant lying above and to the right of another represents a higher output level.* In Figure 29.3 combination B on IQ_1 curve shows larger output than point A on the curve IQ. The combination of OC of capital and OL of labour yields 100 units of product while OC_1 of capital and OL_1 of labour produces 200 units. Therefore, the isoquant IQ_1 which lies above and to the right of the isoquant IQ represents a larger output level.

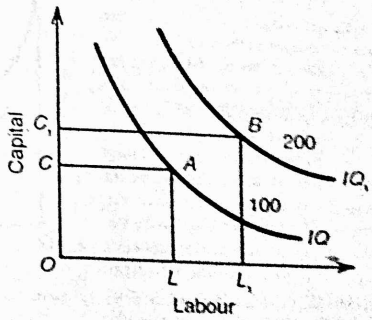

Fig. 29.3

(3) *No two isoquants can intersect each other.* The absurd conclusion that follows when two isoquants cut each other is explained with the aid of Figure 29.4. On the isoquant IQ, combination $A=B$. And on the isoquant IQ_1 combination $R=S$. But combination S is preferred to combination B, being on the higher portion of isoquant IQ_1. On the other hand, combination A is preferred to R, the former being on the higher portion of the isoquant IQ. To put it algebraically, it means that $S > B$ and $R < A$. But this is logically absurd because S combination is as productive as R and A combination produces as much as B. Therefore, the same combination cannot both be less and more productive at the same time. Hence two isoquants cannot intersect each other.

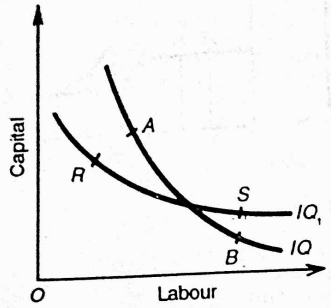

Fig. 29.4

(4) Isoquants need not be parallel because the rate of substitution between two factors is not necessarily the same in all the isoquant schedules.

(5) In between two isoquants there can be a number of isoquants showing various levels of output which the combinations of the two factors can yield. In fact, in between the units of output 100, 200, 300, etc. represented on isoquants there can be innumerable isoquants showing 120, 150, 175, 235, or any other higher or lower unit.

(6) Units of output shown on isoquants are arbitrary. The various units of output such as 100, 200, 300, etc., shown in an isoquant map are arbitrary. Any units of output such as 5, 10, 15, 20 or 1000, 2000, 3000, or any other units can he taken.

(7) *No isoquant can touch either axis.* If an isoquant touches X-axis, it would mean that the product is being produced with the help of labour alone without using capital at all. This is a logical absurdity for OL units of labour alone are incapable of producing anything. Similarly, OC units of capital alone cannot produce anything without the use of labour. Therefore IQ and IQ_1 cannot be isoquants, as shown in Figure 29.5.

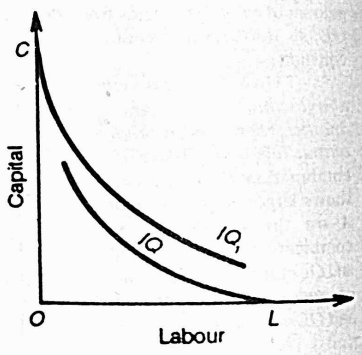

Fig. 29.5

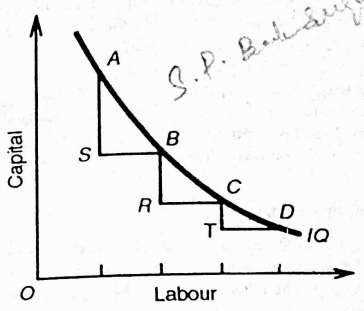

Fig. 29.6

(8) *Each isoquant is convex to the origin.* As more units of labour are employed to produce 100 units of the product, lesser and lesser units of capital are used. This is because the marginal rate of substitution between two factors diminishes. In Figure 29.6, in order to produce 100 units of the product, as the producer moves along the isoquant from combination A to B and to C and D, he gives up smaller and smaller units of capital for additional units of labour. To maintain the same output of 100 units, BR less of capital and relatively RC more of labour is used. If he were producing this output with the combination D, he would be employing CT less of capital and relatively TD more of labour. Thus the isoquants are convex to the origin due to diminishing marginal rate of substitution. This fact becomes clear from successively smaller triangles below the IQ curve $\Delta ASB > \Delta BRC > \Delta CTD$.

(9) Each isoquant is oval-shaped. It is elliptical which means that at some point it begin to recede from each axis. This shape is a consequence of the fact that if a producer uses more of capital or more of labour or more of both than is necessary, the total product will eventually decline. The firm will produce only in those segments of the isoquants which are convex to the origin and lie between the ridge lines. This is the economic region of production. In Figure 29.7, oval-shaped isoquants are shown. Curves OA and OB are the ridge lines and in between them

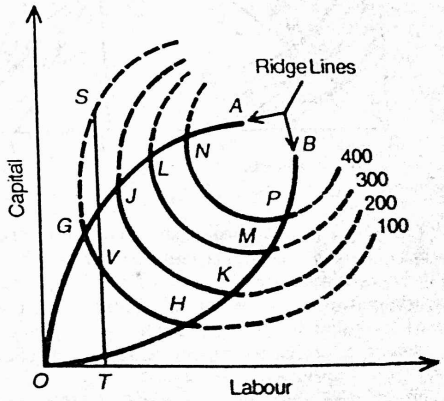

Fig. 29.7

economically feasible units of capital and labour can be employed to produce 100, 200, 300 and 400 units of the product. For example, *OT* units of labour and *ST* units of the capital can produce 100 units of the product, but the same output can be obtained by using the same quantity of labour *OT* and less quantity of capital *VT*. Thus only an unwise entrepreneur will produce in the dotted region of the iso-quant 100. The dotted segments of an isoquant are the *waste-bearing segments*. They form the *uneconomic regions of production*. In the upper dotted portion, more capital and in the lower dotted portion more labour than necessary is employed. Hence *GH, JK, LM,* and *NP* segments of the elliptical curves are the isoquants.

...ther and ...diminishes as labour is substituted for capital. Thus, the marginal rate of ...AS. Thus, the marginal rate of the isoquant must be convex to the origin at every point.

4. THE LAW OF VARIABLE PROPORTIONS

The behaviour of the law of variable proportions or of the short-run production function when one factor is constant and the other variable, can also be explained in terms of the isoquant analysis[5]. Suppose capital is a fixed factor and labour is a variable factor. In Figure 29.10., *OA* and *OB* are the ridge lines and it is in between them that economically feasible units of labour and capital can be employed to produce 100, 200, 300, 400 and 500 units of output. It implies that in these portions of the isoquants, the marginal product of labour and capital is positive. On the other hand, where these ridge lines cut the isoquants the marginal product of the inputs is zero. For instance, at point *H* the marginal product of capital is zero, and at point *L* the marginal product of labour is zero. The portion of the isoquant that lies outside the ridge lines, the marginal product of that factor is negative. For instance, the marginal product of capital is negative at *G* and that of labour at *R*.

The law of variable proportions says that, given the technique of production, the application of more and more units of a variable factor, say labour, to a fixed factor, say capital, will, until a certain point is reached, yield more than proportional increases in output, and thereafter less than proportional increases in output. Since the law refers to increases in output, it relates to the marginal product. To explain the law, capital is taken as a fixed factor and labour as a variable factor. The isoquants

5. Students can explain this law and the Law of Returns to Scale with either of the approaches, the traditional or the modern. But the statement, assumptions and causes must be given.