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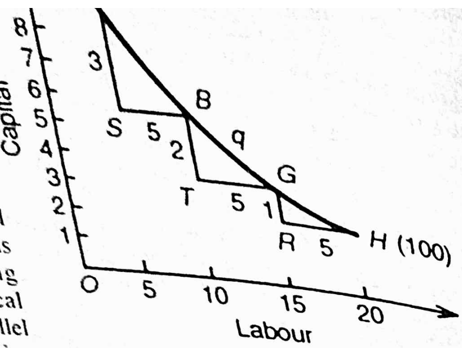

Fig. 29.9

inal rate of substitution in the indifference curve technique. This tendency of nishing marginal substitutability of factors is apparent from Table 29.2 and ure 29.9. The $MRTS_{LC}$ continues to decline from 3:5 to 1:5 whereas in the Figure 9 the vertical lines below the triangles on the isoquant become smaller and naller as we move downward so that $GR < BT <$ AS. Thus, the marginal rate of echnical substitution diminishes as labour is substituted for capital. It means that he isoquant must be convex to the origin at every point.

4. THE LAW OF VARIABLE PROPORTIONS

The behaviour of the law of variable proportions or of the short-run production function when one factor is constant and the other variable, can also be explained in terms of the isoquant analysis[5]. Suppose capital is a fixed factor and labour is a variable factor. In Figure 29.10., *OA* and *OB* are the ridge lines and it is in between them that economically feasible units of labour and capital can be employed to produce 100, 200, 300, 400 and 500 units of output. It implies that in these portions of the isoquants, the marginal product of labour and capital is positive. On the other hand, where these ridge lines cut the isoquants the marginal product of the inputs is zero. For instance, at point *H* the marginal product of capital is zero, and at point *L* the marginal product of labour is zero. The portion of the isoquant that lies outside the ridge lines, the marginal product of that factor is negative. For instance, the marginal product of capital is negative at *G* and that of labour at *R*.

The law of variable proportions says that, given the technique of production, the application of more and more units of a variable factor, say labour, to a fixed factor, say capital, will, until a certain point is reached, yield more than proportional increases in output, and thereafter less than proportional increases in output. Since the law refers to increases in output, it relates to the marginal product. To explain the law, capital is taken as a fixed factor and labour as a variable factor. The isoquants

5. Students can explain this law and the Law of Returns to Scale with either of the approaches, the traditional or the modern. But the statement, assumptions and causes must be given.

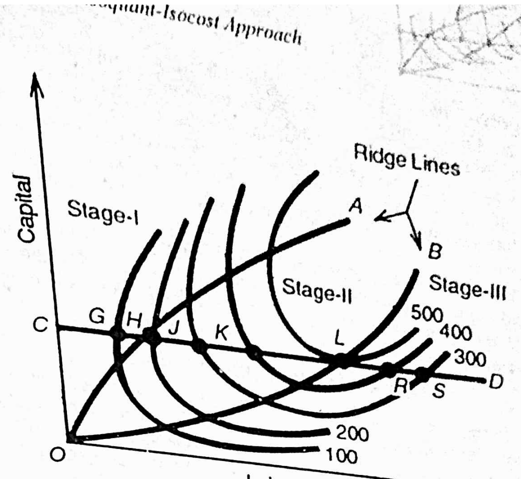

Fig. 29.10

show different levels of output in the figure. *OC* is the fixed quantity of capital which therefore forms a horizontal line *CD*. As we move from *C to D* towards the right on this line, the different points show the effects of the combinations of successively increasing quantities of labour with fixed quantity of capital *OC*.

To begin with, as we move from *C to G to H*, it shows the first stage of *increasing marginal returns* of the law of variable proportions. When *CG* labour is employed with *OC* capital, output is 100. To produce 200 units of output, labour is increased by *GH* while the amount of capital is fixed at *OC*. The output has doubled but the amount of labour employed has not increased proportionately. It may be observed that $GH < CG$, which means that smaller additions to the labour force have led to equal increment in output. Thus *C* to *H* is the first stage of the law of variable proportions in which the marginal product increases because output per unit of labour *increases* as more output is produced.

The *second* stage of the law of variable proportions is the portion of the isoquants which lies in between the two ridge lines *OA* and *OB*. It is the stage of *diminishing marginal returns* between points H and L. As more labour is employed, output increases less than proportionately to the *increase* in the labour employed. To raise output to 300 units from 200 units, *HJ* labour is employed. Further, *JK* quantity of labour is required to raise output from 300 to 400 and *KL* of labour to raise output from 400 to 500 . So, to increse output by 100 units successively, more and more units of the variable factor (labour) are required to be applied along with the fixed factor (capital), that is $KL>JK>HJ$. It implies that the marginal product of labour continues to decline with the employment of larger quantities to it. Thus as we more from point *H* to *K*, the effect of increasing the units of labour is that output per units of labour diminishes as more output is produced. This is known as the stage of diminishing returns.

If labour is employed further, we are outside the lower ridge line *OB* and enter the *third* stage of the law of variable proportions. In this region which lies beyond the ridge line *OB* there is too much of the variable factor (labour) in relation to the fixed factor (capital). Labour is thus being overworked and its marginal product is negative. In other words, when the quantity of labour is increased by *LR* and *RS*, the output declines from 500 to 400 and to 300. This is the stage of *negative marginal returns*.

We arrive at the conclusion that a firm will find it profitable to produce only in the second stage of the law of variable proportions for it will be uneconomical to produce in the regions to the left or right of the ridge lines which form the first stage and the third stage of the law respectively.

5. THE LAWS OF RETURNS TO SCALE

The laws of returns to scale can also be explained in terms of the isoquant approach. The laws of returns to scale refer to the effects of a change in the scale of factors (inputs) upon output in th long-run when the combinations of factors are changed in some proportion. If by incresing two factors, say labour and capital, in the same proportion, output increases in exactly the same proportion, there are constant returns to scale. If in order to secure equal increases in output, both factors are increased in larger proportionate units, there are decreasing returns to scale. If in order to get equal increases in output, both factors are increased in smaller proportionate units, there are increasing returns to scale.

The returns to scale can be shown diagrammatically on an expansion path "by the distance between successive 'multiple-level-of-output' isoquants; that is, isoquants that show levels of output which are multiples of some base level of output, e.g., 100, 200, 300, etc."[6]

Increasing Returns to Scale

Figure 29.11 shows the case of increasing returns to scale where to get equal increases in output, lesser proportionate increases in both factors, labour and capital, are required. It follows that in the figure

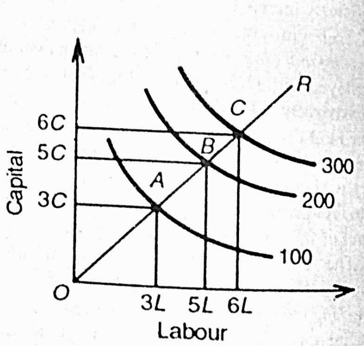

Fig. 29.11

100 units of output require 3C + 3L
200 units of output require 5C + 5L
300 units of output require 6C + 6L

so that along the expansion path *OR*, *OA* > AB > *BC*. In this case, the production function is homogeneous of degree *greater* than one.

6. A. Koutsoyiannis, *op. cit.*, p.79.

The increasing returns to scale are attributed to the existence of indivisibilities in machines, management, labour, finance, etc. Some items of equipment or some activities have a minimum size and cannot be divided into smaller units. When a business unit expands, the returns to scale increase because the indivisible factors are employed to their full capacity.

Increasing returns to scale also result from specialisation and division of labour. When the scale of the firm expands there is wide scope for specialisation and division of labour. Work can be divided into small tasks and workers can be concentrated to narrower range of processes. For this, specialized equipment can be installed. Thus with specialization, efficiency increases and increasing returns to scale follow.

Further, as the firm expands, it enjoys internal economies of production. It may be able to install better machines, sell its products more easily, borrow money cheaply, procure the services of more efficient manager and workers, etc. All these economies help in increasing the returns to scale more than proportionately.

Not only this, a firm also enjoys increasing returns to scale due to external economies. When the industry itself expands to meet the increased long-run demand for its product, external economies appear which are shared by all the firms in the industry. When a large number of firms are concentrated at one place, skilled labour, credit and transport facilities are easily available. Subsidiary industries crop up to help the main industry. Trade journals, research and training centres appear which help in increasing the productive efficiency of the firms. Thus these external economies are also the cause of increasing returns to scale.

Decreasing Returns to Scale

Figure 29.12 shows the case of decreasing returns where to get equal increases in output, *larger* proportionate increases in both labour and capital are required. It follows that

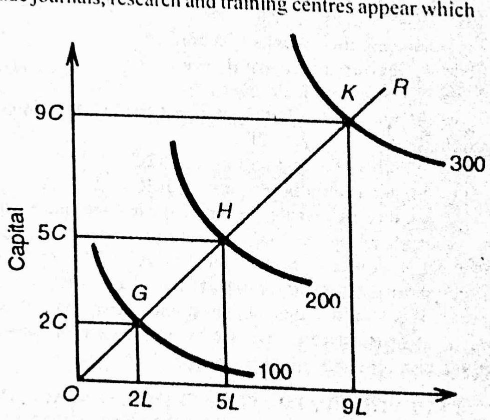

Fig. 29.12

100 units of output require 2C + 2L
200 units of output require 5C + 5L
300 units of output require 9C + 9L

so that along the expansion path *OR*, *OG* < *GH* < *HK*.

In this case, the production function is homogeneous of degree *less* than one.

Returns to scale may start diminishing due to the following factors. Indivisible factors may become inefficient and less productive. Business may become unwieldy and produce problems of supervision and coordination. Large management creates difficulties of control and rigidities. To these internal diseconomies are added external diseconomies of scale. These arise from higher factor prices or from diminishing productivities of the factors. As the industry continues to expand the demand for skilled labour, land, capital, etc. rises. There being perfect competition, intensive bidding raises wages, rent and interest. Prices of raw materials also go up. Transport and marketing difficulties emerge. All these factors tend to raise costs and the expansion of the firms leads to diminishing returns to scale so that doubling the scale would not lead to doubling the output.

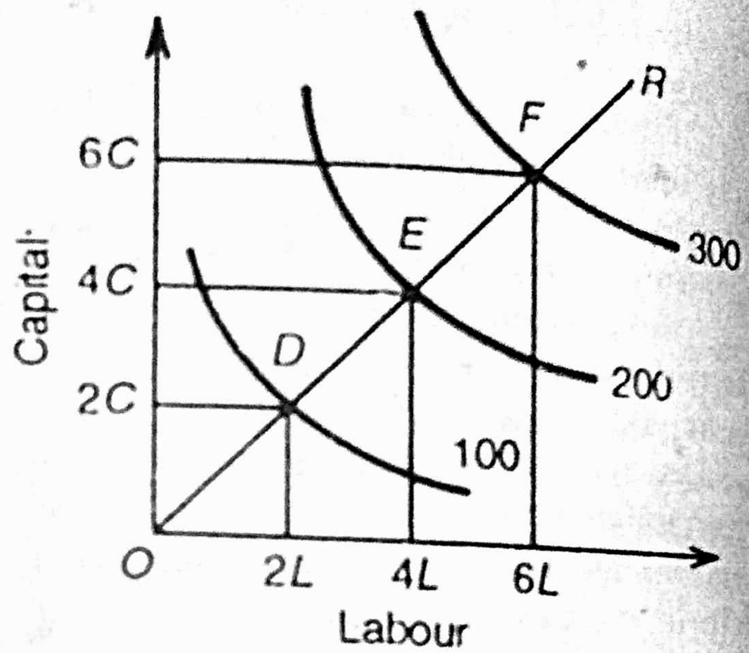

Fig. 29.13

Constant Returns to Scale

Figure 29.13 shows the case of constant returns to scale. Where the distance between the isoquants 100, 200 and 300 along the expansion path *OR* is the same, *i.e.*, *OD* = *DE* = *EF*. It means that if units of both factors, labour and capital, are doubled, the output is doubled. To treble output, units of both factors are trebled. It follows that 100 units of output require 1 (2C + 2L) = 2C + 2L

200 units of output require 2(2C + 2L) = 4C + 4L

300 units of output require 3(2C + 2L) = 6C + 6L

The returns to scale are constant when internal economies enjoyed by a firm are neutralised by internal diseconomies so that output increases in the same proportion. Another reason is the balancing of external economies and external diseconomies. Constant returns to scale also result when factors of production are perfectly divisible, substitutable, homogeneous and their supplies are perfectly elastic at given prices. That is why, in the case of constant returns to scale, the production function is homogeneous of degree one.

6. RELATION BETWEEN RETURNS ...

scale, the marginal returns of the variable factor increase instead of diminishing.

7. CHOICE OF OPTIMAL FACTOR COMBINATION OR LEAST COST COMBINATION OF FACTORS OR PRODUCER'S EQUILIBRIUM

A Profit maximisation firm faces two choices of optimal combination of factors (inputs): First, to minimise its cost for a given output; and second, to maximise its output for a given cost. Thus the least cost combination of factors refers to a firm producing the largest volume of output from a given cost and producing a given level of output with the minimum cost when the factors are combined in an optimum manner. We study these cases separately.

Cost-Minimisation for a Given Output

In the theory of production, the profit maximisation firm is in equilibrium when, given the cost price function, it maximises its profits on the basis of the least cost combination of factors. For this, it will choose that combination which minimises its cost of production for a given output. This will be the optimal combination for it.

Its Assumptions

This analysis is based on the following assumptions:

1. There are two factors, labour and capital.
2. All units of labour and capital are homogeneous.
3. The prices of units of labour *(w)* and that of capital *(r)* are given and constant.
4. The cost outlay is given.
5. The firm produces a single product.
6. The price of the product is given and constant.
7. The firm aims at profit maximisation.
8. There is perfect competition in the factor-market.

Given these assumptions, the point of least-cost combination of factors for a given level of output is where the isoquant curve is tangent to an isocost line. In Figur 29.17, the isocost line *GH* is tangent to the isoquant 200 at point *M*. The firm employs the combination of *OC* of capital and *OL* of labour to produce 200 units of output at point *M* with the given cost-outlay *GH*. At this point, the firm is minimising its cost for producing 200 units. Any other combination on the isoquant 200, such as *R* or *T*, is on the higher isocost line *KP* which shows higher cost of production. The isocost line *EF* shows lower cost but output 200 cannot be attained with it. Therefore, the firm will choose the minimum cost point *M* which is the least-cost factor combination for producting 200 units of output. *M* is thus the optimal combination for the firm.

The point of tangency between the isocost line and the isoquant is an important first order condition but not a necessary condition for the producer's equilibrium. There are *two essential or second order conditions* for the equilibrium of the firm.

1. The *first condition* is that the slope of the isocost line must equal the slope of the isoquant curve. The slope of the isocost line is equal to the ratio of the price of labour (*w*) to the price of capital (*r*). The slope of the isoquant curve is equal to the marginal rate of technical substitution of labour and capital ($MRTS_{LC}$) which is, in turn, equal to the ratio of the marginal product of labour to the marginal product of capital (MP_L/MP_C). Thus the equilibrium condition for optimality can be written as:

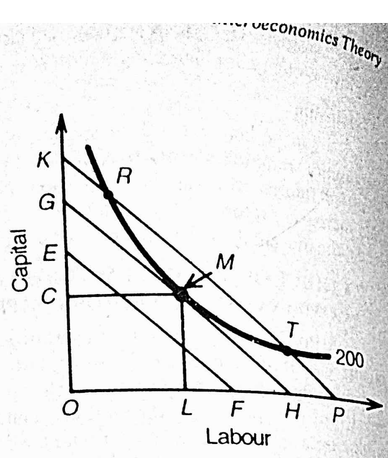

Fig. 29.17

$$\frac{w}{r} = \frac{MP_L}{MP_C} = MRTS_{LC}$$

2. The *second condition* is that at the point of tangency, the isoquant curve must be convex to the origin. In other words, the marginal rate of technical substitution of labour for capital ($MRTS_{LC}$) must be diminishing at the point of tangency for equilibrium to be stable. In Figure 29.18, *S* cannot be the point of equilibrium, for the isoquant IQ_1 is concave where it is tangent to the isocost line *GH*. At point *S*, the marginal rate of technical substitution between the two factors increases if move to the right or left on the curve IQ_1. Moreover, the same output level can be produced at a lower cost *CD* or *EF* and there will be a corner solution either at *C* or *F*. If it decides to produce at *EF* cost, it can produce the entire output with only *OF* labour. If, on the other hand, it decides to produce at a still lower cost *CD*, the entire output can be produced with only *OC* capital. Both the situations are impossibilities because nothing can be produced either with only labour or only capital. Therefore,

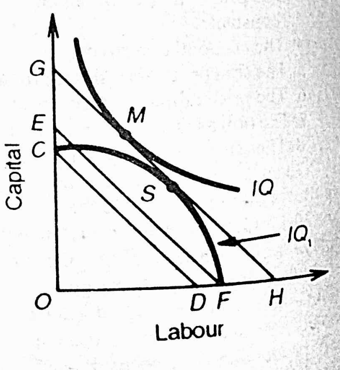

Fig. 29.18

the firm can produce the same level of output at point *M* where the isoquant curve *IQ* is convex to the origin and is tangent to the isocost line *GH*. The analysis assumes that both the isoquants represent equal level of output, $IQ = IQ_1$.

Output-Maximisation for a Given Cost

The firm also maximises its profits by maximising its output, given its cost outlay and the prices of the two factors. This analysis is based on the same assumptions, as given above. The conditions for the equilibrium of the firm are the same, as discussed above.[8]

1. The firm is in equilibrium at point *P* where the isoquant curve 200 is tangent to the isocost line *CL* in Figure 29.19. At this point, the firm is maximising its output level of 200 units by employing the optimal combination of *OM* of capital and *ON* of labour, given its cost outlay *CL*. But it cannot be at points *E* or *F* on the isocost line *CL*, since both points give a smaller quantity of output, being on the isoquant 100, than on the isoquant 200. The firm can reach the optimal factor combination level of maximum output by moving along the isocost line *CL* from either point *E* or *F* to point *P*. This movement involves no extra cost because the firm remains on the same isocost line. The firm cannot attain a higher level of output such as isoquant 300 because of the cost constraint. Thus the equilibrium point has to be *P* with optimal factor combination *OM* + *ON*. At point *P*, the slope of the isoquant curve 200 is equal to the slope of the isocost line *CL*. It implies $w/r = MP_L/MP_C = MRTS_{LC}$.

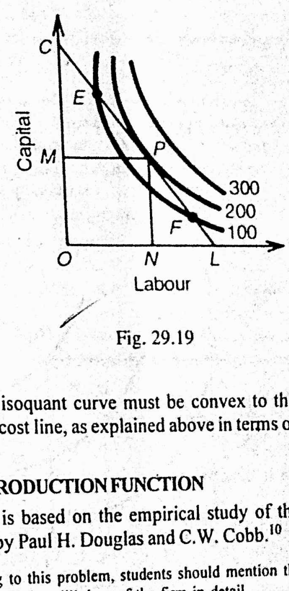

Fig. 29.19

2. The second condition is that the isoquant curve must be convex to the origin at the point of tangency with the isocost line, as explained above in terms of Figure 29.18[9].

8. THE COBB-DOUGLAS PRODUCTION FUNCTION

The Cobb-Douglas production function is based on the empirical study of the American manufacturing industry made by Paul H. Douglas and C.W. Cobb.[10] It

8 If there is an independent question relating to this problem, students should mention the assumptions here and also the two conditions of equilibrium of the firm in detail.

9. Give Figure 29.18 here alongwith its explanation in case of an independent question. Students should note the similarities in the consumer's equilibrium and producer's equilibrium. Except for the objectives that a consumer aims at maximisation of satisfaction and a producer at maximisation of profits, the first order and second order conditions are the same.

10 C.W Cobb and P.H. Douglas, "A Theory of Production," *A.E.R.* (Supplement), 1928. For the meaning of production function, refer to the previous chapter.