MATHEMATICAL DEFINITIONS OF POINT, LINE AND AREA SYMBOLS IN CARTOGRAPHY[*]

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ABSTRACT

Map is the symbolic model of the objective world where there are the variety and variability of things and facts. The line cartographic symbol located on the boundary and the point cartographic symbol located by single point are the main body and gravity of the map. The cartographic symbols are divided point, line and area symbol in cartography and traditional educational books, but there is qualitative description about the three kinds of symbols. Up to now there are not the mathematical definitions of point, line and area symbols. This paper wants to solve this problem. In this paper the authors raise the mathematical definition that point, line and area cartographic symbol according to the relative theory of topology.

1. point cartographic symbol

   There exist point $C(x,y)$ of coordinate in the plane, if set U_i suffices:
   $$U_i = \{P|C(x,y) \in U_i \wedge U_i = f(i)\}$$
   then U_i is called point cartographic symbol.

2. Line cartographic symbol

   After definition of neighborhood and boundary were given by the topology, we may definite line cartographic symbol:
   The cartographic symbol that indicates the i class linear body and with concern boundary as its location line is called the line cartographic symbol. i.e. suffice
   $$l_i = \{P|BdA \in l_i \wedge l_i = f(i), P \in l_i\}$$
   then l_i is called line cartographic symbol.

3. Area cartographic symbol

   There exist connective set $A_j \in X$, and its indicated body with BdA_j as its boundary, i.e. suffice.
   $$A_j = \{P|P \in A_j \wedge A_j = f(j)\}$$
   then A_j is called area cartographic symbol.

The figure is composed of point and line cartographic symbol and it's the main body of map, the ground is composed of area cartographic symbol. The figure and ground bring into play whole function in the transmission of geographical information.

Infinite complicated objects are reflected and showed on the limit maps by the multi-thematic and multi-scale maps. Finite things and phenomena connected with themes always occupy the whole cartographic region and form the structure of either this or that and connect with each other, which reflects the geographic character. There is the visible or invisible boundary between things and phenomena of various features in cartographic region. The boundary not only indicates the borders of the objects, but also effective reflect the objects of the borders. Therefore the line cartographic symbols be located at the boundary and the point cartographic symbols be located at single point are the main body and stress of the map. The cartographic symbols are divided point, line and area symbol in cartography and traditional educational books, but there is qualitative description about the three kinds of symbols. Up to now there are not the mathematical definitions of point、line and area symbols. This paper wants to solve this problem.

Keywords: point cartographic symbol; line cartographic symbol; area cartographic symbol; figure

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1. POINT CARTOGRAPHICAL SYMBOL

Definite coordinate point C (x, y) is oriented and indicated the feature and character or some quantitative value of the objects of this point, which are important attributes of point cartographic symbol. The point cartographic symbols connect with more visual variables such as shape、size、direction、structure、color and so on[1]. So the point cartographic symbol belongs to the point set category.

Definition 1: There exists planar coordinate point C (x, y), if the condition as following:
$$U_i = \{P|C(x, y) \in U_i \wedge U_i = f(i)\}. \tag{1}$$
is satisfied, then point set U_i is called the point cartographic symbol which is fixed on C (x, y) and indicates the feature i.

There are more kinds of fixed means of point cartographic symbol (figure.1). In various fixed means the position of fixed point C (x, y) in U_i is different. But these suffices the condition of C (x, y) $\in U_i$. $U_i = f(i)$ indicates that U_i is the mapping of feature i in the map.
(1)is the common equation of point cartographic symbols such as triangle point, tower building, transformer substation, single tree and so on.
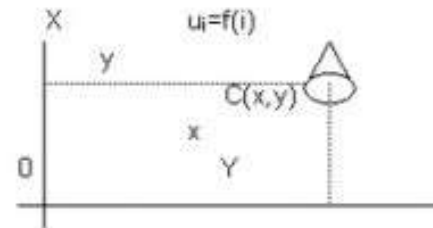

Figure 1. The Sketch of Point Cartographic Symbol

2. LINE CARTOGRAPHIC SYMBOL

There exist the borderlines among different characteristic and different extent features. The line cartographic symbol is fixed on the borderline and indicates the function of the feature, so we must definite the neighborhood and boundary (Figure 2).

2.1 Neighborhood
Definition 2: Let a be a point in metrical space X and N_a be a point set in X, if the condition as following:
$$\delta > 0 \supset B(a, \delta) \subset N_a$$
is satisfied, then N_a is called a-neighborhood in X.

a - neighborhood $B(a, \delta)$ with radius of δ
open a-neighborhood $N_a = U_A \cup U_{X-A}$
the boundary of A and X - A BdA
The line cartographic symbol Li

2.2 Boundary
Definition 3: Let A be a subset of topological space X, $a \in X$, if an arbitrary a-neighborhood has points in both A and $X - A$, then we call a is a bounding point of A. The set of all bounding points is called the boundary of A, and is denoted by BdA[3].

Figure 2. The Relation of Neighborhood, Boundary and Line Cartographic Symbol

$$BdA = \{a| a \in Na = U_A \cup U_{X-A}, U_A \in A, U_{X-A} \in X-A\} \tag{2}$$

2.3 The line cartographic symbol
Definition 4: The cartographic symbol fixed on relative boundary is called line cartographic symbol which indicates the line feature i. It suffices
$$l_i = \{P | BdA \in l_i \wedge l_i = f(i), P \in l_i\} \tag{3}$$
The line cartographic symbol li is the point set. BdA in (3) is provided by (2), and i is feature indicated by line cartographic symbol. (3) is the common equation of the line cartographic and so on.

3. AREA CARTOGRAPHIC SYMBOL

3.1 Connected set
Definition 5: If there exists a point set $A \in X$, A can not be expressed by : sets, namely, there can not exist open subsets U, V of A, $U \neq \phi, V \neq$ connected set[2].
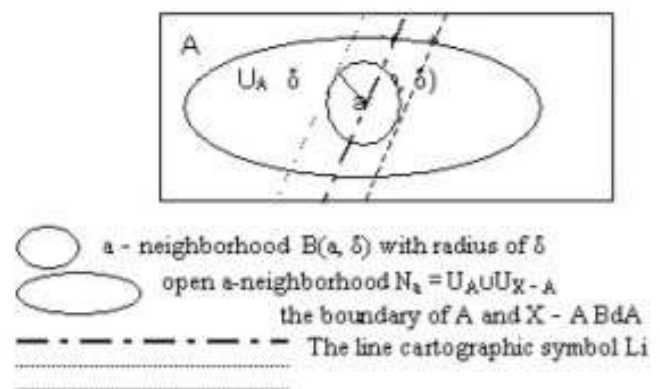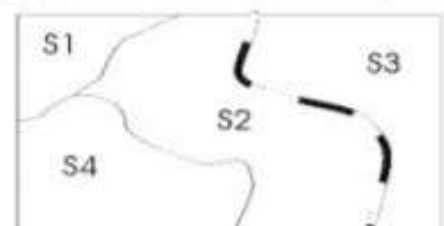

G is composed of inside skeleton map, single river, bank line and railway.
$$S = X - G = G^c = S_1 \cup S_2 \cup S_3 \cup S_4$$

Figure 3. The Sketch of Area Cartographic Symbol

3.2 Area cartographic symbol

Definition 6: If there exists connected set $A_j \in X$, which indicates the attribute of feature besieged by the boundary BdA_j of A_j, then point set A_j is called the area cartographic symbol and it suffices:

$$A_j = \{P|P \in A_j \wedge A_j = f(j)\} \tag{4}$$

A_j in (4) is provided by definition 5, and j is feature indicated by area cartographic symbol. Considering the different essence of point, line and area cartographic symbol, we promise that i indicates point and line feature, and that j indicates area feature. (4) is the common equation of area cartographic symbol such as water field, woodland, plowland etc. Because the connected set has completed boundary, it can describe any area field.

4. FIGURE AND BACKGROUND OF MAP

4.1 Point and line cartographic symbols are the main body of map

That point and line cartographic symbols are the main body of map reflects the following aspects:

- Map is the subset of graphic set. Satisfying the definition of graph is the basic condition and logical demand of cartographic definition [4], and the set of point and line is the main condition of constructing graph [5].
- Bearing the weight of map is composed of symbols and area of annotation of the map elements of residential area, waterbody, road, border and so on [6], i.e. bearing the weight of map is composed of point and line cartographic symbols and area of annotation. Bearing the weight of map is connected with map legibility and its quality [7].
- Point and line cartographic symbols lie always higher visual level in the map. The important point and line symbols are more than prominence.
- Constructing graph of point and line symbols is the comparative standard of distinguishing moon, sun, positive, negative of the map figure [8].

Constructing graph of map main elements is expressed by the combined set of the set of point cartographic symbol and the set of line cartographic symbol, i.e.

$$G = \{U_i\} \cup \{l_i\}|i \in I \tag{5}$$

4.2 Area cartographic symbol constructs the background of map

Definition 7: The set composed by whole area cartographic symbol or the complemented set of the set of point and line cartographic symbols is called the background of map, that is,

$$S = \cup A_j = X - G = G^c \tag{6}$$
$$j \in J$$

In (6) Aj is provided by (4) and G is provided by (5).

Any map is always the unite body of figure and background. Formular (6) expresses this relation of opposite and union.

4.3 The unite function of figure and background in map information transmission

The map is the symbolic model of object world. The research shows the information in map is more abundant than that in general books and newspaper [9]. In these days basing on inexperienced statistical way a general map can contain and save the information of 100-200 million informative units [10]. Therefore, as the media and bridge people's recognizing object world, map is more intuitional and more effective than other means. Though the figure and background lie different visual level, they exert unit function in transmitting geographic information for their interdependent relation. They react supplemental each other function on reflecting geographic character. ①Unformed or inconspicuous form borderlines in the object world are prominent for the structure of point and line symbols, and the area symbol is prominent for the prominent borderline and the difference of borderline and area symbol in vision. ②Different characteristic point and line symbols differentiate each other for tint area symbol, thus various point and line symbols are effective reflected in the map. Especially, recessive things (such as air temperature, border, population) are displayed and emphasized by cartographic symbols and we can survey all through the map. All those show the superiority of the map media.

5. CONCLUSION

Based on the theory of set and topology, combining the common attribute and character of cartographic symbols, we definite the point, line and area cartographic symbols in mathematic way, which makes confined qualitative description cartographic symbols obtain exact mathematic form and quantitative description. The interdependent and opposite and union relations of point, line and area cartographic symbols make them exert the important effect and unite function in the transmission of geographic information.

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